MULTIUSER DETECTION: AN OVERVIEW AND A NEW RESULT

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ABSTRACT

The purpose of this paper is to review some important models and methods in receiver design for multiuser communication, and to outline some open problems of potential interest to researchers with a signal processing and information/communication theory background. We also present a new interference cancellation technique that permits operation in an oversaturated regime, in which the number of users exceeds the number of signaling dimensions.

1. INTRODUCTION

Multiuser detection, or interference suppression, addresses the problem of reliable demodulation in the presence of the multiple-access interference (MAI) created by the reception of signals from many simultaneous users. While there are a variety of fundamental theoretical challenges in this area, it has received much recent attention because of its potential technological impact on wireless multiuser systems, such as digital cellular telephony based on direct sequence (DS) code division multiple access (CDMA). The purpose of this paper is to provide a selective overview of recent developments and open problems in multiuser detection that are likely to be of interest to the attendees of this workshop. These include a new interference cancellation technique termed Parallel Arbitrated Serial Interference Cancellation (PASIC), which in preliminary evaluations offers significant gains over previous interference cancellation schemes.

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Our goal in this paper is to convey the mathematical essentials of some multiuser detection problems to a reader with a sophisticated signal processing background. The reader who wishes to supplement this with a more thorough introduction to the subject is referred to the recent book on multiuser detection by one of the pioneers in the field [29], and to a tutorial [17] on blind adaptive interference suppression for CDMA systems by the author of the present paper. Note that the references in the paper are a somewhat random sampling of the literature, and are not meant to provide a comprehensive survey. We apologize in advance to the authors of the many important papers that have been left out of the bibliography.

The paper is organized as follows. A generic model for a multiuser system is presented in Section 2. Several special cases of the model are discussed. An overview of multiuser detection strategies is given in Section 3. These include centralized receivers that require explicit knowledge of the paramters of all users, and adaptive receivers that implicitly learn the structure of the MAI, starting only with some knowledge regarding the desired user. The modifications required to deal with time-varying channels, and the interplay between multiuser detection and decoding are also discussed. The new PASIC detector is described in Section 4, and some open problems are sketched in Section 5.

2. MODELING MULTIUSER SYSTEMS

The following general model for linearly modulated multiuser systems is specified in complex baseband. Assuming that all users employ linear modulation, the L-dimensional complex received vector of samples from the nth observation interval has the form

$$\mathbf{r}[n] = \sum_{j=1}^{K} d_j[n]\mathbf{u}_j[n] + \mathbf{w}[n] = \mathbf{U}[n]\mathbf{d}[n] + \mathbf{w}[n] \quad (1)$$

where, for $1 \leq j \leq K$, $\{d_j[n]\}$ denote data streams such that $d_j[n]$ and $d_k[n]$ are uncorrelated for $j \neq k$, and $\mathbf{d}[n] = (d_1[n], ..., d_K[n])^T$. [1] For $1 \leq j \leq K$, the signal vector $\mathbf{u}_j[n]$ is a possibly time-varying signal modulated by the jth data stream, and the matrix $\mathbf{U}[n]$ contains as columns these signal vectors. The vector $\mathbf{w}[n]$ is noise, modeled as complex white Gaussian with variance σ^2 per dimension. This is identical to the standard model used in independent component analysis, except that some specific information may be available regarding the signal vectors $\{\mathbf{u}_j[n]\}$, and that some explicit time variations in these vectors (due to, say, a wireless mobile channel) may need to be accounted for. Our convention for numbering symbols is such that $\mathbf{r}[n]$ is used for demodulating (a subset of the symbols in) $\mathbf{d}[n]$. Thus, the observation intervals corresponding to $\mathbf{r}[n]$ and $\mathbf{r}[n+1]$ are offset by the inverse of the data rate.

A centralized multiuser detection algorithm would typically assume that explicit estimates of the signal vectors $\{\mathbf{u}_j[n]\}$ are available. On the other hand, when the variations in the vectors $\{\mathbf{u}_j[n]\}$ are either slow (as a function of n), or can be captured by means of a set of time-varying parameters whose size is small compared to the size of the parameter space required to specify the vectors themselves, we can hope to apply *adaptive* interference suppression methods which do not require explicit estimates of the signal vectors. Instead, implicit information about the signal vectors can be obtained from empirical statistics derived from the sequence of received vectors $\{\mathbf{r}[n]\}$.

The canonical multiuser system we will keep in mind is a linearly modulated DS-CDMA system, in which the symbol sequences of different users modulate different signaling, or *spreading,* waveforms whose bandwidth is significantly larger than the Nyquist bandwidth for each user. In principle, this provides the redundancy needed for the receiver to separate the signals of different users. The following distinction needs to be drawn within the class of DS-CDMA systems:

Long spreading waveforms: the spreading waveforms for different symbols of the same user are different.

Short spreading waveforms: the spreading waveforms for different symbols of the same user are the same. The received impulse response modulated by different symbols of the same user may still differ because of channel time variations.

[1] Since several symbols from the same user may have nonzero response within the observation interval, some of the data sequences may be shifted versions of each other. Thus, it is possible that $d_j[n]$ and $d_j[m]$ are correlated for $j \neq k$ and $n \neq m$.

2.1. Symbol level model

Here the data streams $\{d_j[n]\}$ have a direct interpretation as the information symbols sent by the users. The generic model (1) arises in these systems after filtering, sampling at a multiple of the system bandwidth, and restricting attention to a specific observation interval of samples $\mathbf{r}[n]$ for the nth symbol decision. Any given vector $\mathbf{u}_j[n]$ is therefore obtained by filtering, sampling, and windowing the cascade of the spreading waveform and the channel for a particular symbol of a given user. Successive observation intervals are offset from each other by the inverse of the symbol rate, and may overlap. See [17] for details of such modeling. If an antenna array is available at the receiver, then the space-time signal vectors $\mathbf{u}_j[n]$ have a dimension equal to the product of the number of antenna elements and the size of the temporal observation interval.

For a discrete multipath channel, we may express a typical signal vector as

$$\mathbf{u}_j[n] = \sum_{l=1}^{L_j} h_{jl}[n]\mathbf{s}_{jl}[n] \qquad (2)$$

where h_{jl} is the time-varying complex gain of the lth resolvable path, and $\mathbf{s}_{jl}$ is a delayed, filtered, sampled, and windowed version of the spreading waveform corresponding to the lth path. For short spreading waveforms, $\mathbf{s}_{jl}[n] \equiv \mathbf{s}_{jl}$, so that the time variations only occur due to the channel. Thus, if the channel time variations are absent or can be handled effectively, a system with short spreading waveforms is amenable to adaptive interference suppression methods.

Due to the excess bandwidth employed, the signaling waveforms for the users in a well-designed system are such that the normalized correlation between the signal vectors $\frac{\langle \mathbf{u}_j[n], \mathbf{u}_k[n] \rangle}{||\mathbf{u}_j[n]|| \; ||\mathbf{u}_k[n]||}$ is small for $j \neq k$. Here $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^H \mathbf{y}$ denotes the standard complex inner product between vectors $\mathbf{x}$ and $\mathbf{y}$, and $||\mathbf{x}|| = \sqrt{\langle \mathbf{x}, \mathbf{x} \rangle}$ denotes the norm of a vector $\mathbf{x}$.

The co-channel and intersymbol interference in narrowband systems follows the same model as a CDMA system with short spreading waveforms. The key distinction is that the normalized correlations can be large, because the excess bandwidth employed is small.

2.2. Chip-level model

Consider now the detailed structure of a typical DS-CDMA system, where the spreading waveforms are obtained by linearly modulating a spreading sequence of complex symbols using a chip waveform. Thus, the transmitted waveform modulating the nth information symbol can be written as

$$s(t;n) = \sum_{m=1}^{N} c[nN+l]\psi(t - nNT_c - lT_c)$$

where $\{c[l]\}$ is the spreading, or chip, sequence, which is complex, in general, $\psi(t)$ is the chip waveform, N is the processing gain, or the number of chips per symbol, and $\frac{1}{T_c}$ is the chip rate (which is N times the symbol rate). The chip waveform determines the bandwidth of the system, and is typically a Nyquist or square root Nyquist pulse at the chip rate. For long spreading waveforms, the chip sequence is aperiodic, or has period much larger than N, whereas for short spreading waveforms, the chip sequence is periodic with period N.

For DS-CDMA with long spreading waveforms, the signal vectors modulating each symbol vary from symbol to symbol, so that adaptive interference suppression does not apply to such systems at the symbol level. However, the required quasi-stationarity does hold at the chip rate. In particular, let n in (1) index the chip time, and set $d_j[n]$ equal to the product of the symbol sequence (piecewise constant over N chip times) and the chip sequence, the signal vector $\mathbf{u}_j[n]$ is the filtered, sampled and windowed version of the cascade of the (time-invariant) chip filter and the (slowly varying) channel. Thus, adaptive interference suppression does apply at the chip rate. Since the excess bandwidth at the chip rate is small, the capability for separating multiple users using such adaptive processing is limited unless the dimension of the received vector is increased in some fashion, e.g., by deploying an antenna array at the receiver. Once the chips have been reliably extracted using such interference suppression mechanisms, they can be combined as in a conventional system to provide decision statistics for the transmitted symbols. The reader is referred to [7] and [18] for examples of chip level interference suppression.

3. MULTIUSER DETECTION STRATEGIES

The ultimate performance measure in communication systems is the error probability. Most of the multiuser detection literature has focused on the uncoded error probability for hard decisions. Further, since computation of the error probability is often difficult in multiuser systems (involving as it does averaging over all possible data realizations among all users), the literature has focused on asymptotic measures such as the *asymptotic efficiency,* which measures the exponent of the decay of the error probability as the noise level vanishes, relative to the exponent when there is no multiple-access interference. Another quantity of interest is the *near-far resistance,* which is the worst-case value of the asymptotic efficiency over all possible choices of the amplitudes of the interference vectors. While much progress has been made by characterizing these asymptotic performance measures for various multiuser detection algorithms, these measures may be somewhat irrelevant for modern communication systems. This is because, with the emergence of powerful error control coding techniques such as turbo and turbo-like codes, it is required that multiuser detection algorithms provide *soft,* rather than hard decisions, to the decoder, and for efficient operation of turbo-like codes, the uncoded bit error probability for hard decisions should be kept in the 10% range, which is much larger than the error probabilities corresponding to the asymptotic regime in which the performance of multiuser detection algorithms has been characterized to date. Recent Shannon-theoretic analyses of multiuser systems with different receivers do address such issues [31], but much work remains in obtaining *practical* receivers (in terms of computational complexity, delay, and robustness) for the design of coded multiuser communication systems.

3.1. Non-adaptive receivers

For non-adaptive receivers, it is assumed that estimates of the signal vectors $\{\mathbf{u}_j[n]\}$ (or at least of the signal vector corresponding to the desired user) are available at the receiver. Note that this means that estimates of the propagation channel, as well as knowledge of the signaling waveforms, are available at the receiver. Restricting attention to processing only within a particular observation interval (i.e., assuming that the possible dependencies among successive observation intervals will not be exploited), we drop the dependence on the time variable n while discussing such strategies.

For detection in additive white Gaussian noise, the *matched filter* sufficient statistics are obtained by correlating the received vector with each of the signal vectors. Recalling that $\mathbf{U}$ denotes the matrix whose columns are the signal vectors $\{\mathbf{u}_j\}$, the vector of sufficient statistics $\mathbf{Z} = (Z_1, ..., Z_K)^T$ is given by [29]

$$\mathbf{Z} = \mathbf{U}^H\mathbf{r} = \mathbf{R}_u\mathbf{d} + \mathbf{n} \tag{3}$$

where $\mathbf{R}_u = \mathbf{U}^H\mathbf{U}$ is the matrix of crosscorrelations between the signal vectors, and $\mathbf{n}$ is colored complex Gaussian noise with covariance matrix $2\sigma^2\mathbf{R}_u$.

Maximum Likelihood (ML) Detection

When the noise in (1) is modeled as white, ML detection [30] consists of minimizing the Euclidean distance between the received vector and the set of possible noiseless received signals. The ML estimate is given by

$$\hat{\mathbf{d}}_{ML} = \arg\min_{\mathbf{d}} ||\mathbf{r} - \mathbf{U}\mathbf{d}||^2 \quad (4)$$

In terms of the matched filter sufficient statistics, ML detection corresponds to maximization of $2Re(\mathbf{d}^H \mathbf{Z}) - \mathbf{d}^H \mathbf{R}_u \mathbf{d}$. In view of the exponentially many possibilities for $\mathbf{d}$, implementation of the ML detector is infeasible for most practical scenarios. The maximum a posteriori (MAP) detector, which minimizes the error probability for each user of interest, is closely related to the ML detector, and has comparable complexity.

Linear Receivers

For a linear receiver, the decision statistic for a given data symbol d_j is given by $\langle \mathbf{c}_j, \mathbf{r} \rangle$, where $\mathbf{c}_j$ is an appropriately chosen correlator. While suboptimal, the class of linear receivers are computationally more attractive than the ML receiver.

Matched filter receiver: This corresponds to $\mathbf{c}_j = \mathbf{u}_j$, so that the decision statistic for d_j is

$$Z_j = \mathbf{R}_u(j,j) d_j + \sum_{k \neq j} \mathbf{R}_u(j,k) d_k + n_j \quad (5)$$

The advantage of this receiver is that it requires knowledge of only the desired signal vector $\mathbf{u}_j$ at the receiver. However, it is clear from (5) that the statistic Z_j can, in general, contain significant contributions from the interfering data symbols d_k, $k \neq j$, so that the performance of the matched filter receiver is interference limited. Moreover, if one of the interference vectors $\mathbf{u}_k$ is significantly stronger (i.e., with more energy) than the vector $\mathbf{u}_j$, then $\mathbf{R}_u(j,k)$ may be of the same order, or larger than, $\mathbf{R}_u(j,j)$, making reliable demodulation of d_j using Z_j alone impossible. This is because the contribution to Z_j of d_k is comparable to, or larger than, that of the desired data d_j. Such a phenomenon is referred to as the *near-far problem,* and is avoided in current DS-CDMA systems such as IS-95 by use of closed loop power control.

Decorrelating, or zero-forcing, receiver : In this receiver [14, 15], the correlator $\mathbf{c}_j$ is chosen along the component of $\mathbf{u}_j$ orthogonal to the subspace $\mathbf{S}_{I,j}$ spanned by the interference vectors $\{\mathbf{u}_k, k \neq j\}$. That is, $\mathbf{c}_j = \mathcal{P}^{\perp}{}_{\mathbf{S}_{I,j}} \mathbf{u}_j$. This receiver exists if, and only if, $\mathbf{u}_j$ is linearly independent of any basis for $\mathbf{S}_{I,j}$. While the projection operation eliminates the interference completely, thus removing the near-far problem, it enhances the noise contribution relative to the signal. The decorrelating detector statistics can be expressed in terms of the matched filter sufficient statistics as follows:

$$\mathbf{Z}_{dec} = \mathbf{R}_u^{-1} \mathbf{Z} = \mathbf{d} + \mathbf{n}_{dec} \quad (6)$$

where $\mathbf{n}_{dec}$ is colored Gaussian noise with covariance matrix proportional to $\mathbf{R}_u^{-1}$. The decision for d_j is made based only on the jth entry of $\mathbf{Z}_{dec}$, which is suboptimal, because the noise samples within $\mathbf{n}_{dec}$ are correlated.

An alternative to the decorrelator is the linear MMSE receiver [36, 19], whose discussion is postponed to Section 3.2 on adaptive receivers, since one of its key advantages is its amenability to adaptive implementation.

Interference Cancellation

The basic idea is to update estimates of a subset of the data by cancelling the interference in the matched filter sufficient statistics based on previous estimates of the data. Let $\hat{\mathbf{d}}^i$ denote the data estimates at stage i. Suppose that at stage $i+1$, the subset $U(i+1) \subset \{1,..,K\}$ of the data is to be updated. Then the decision statistic for data symbol j at the next stage is given by

$$Z_j^{(i+1)} = Z_j - \sum_{k \neq j} R_u(j,k) \hat{d}_k^{(i)}, \quad j \in U(i+1)$$

(For example, for binary antipodal signal, the updated data values after this iteration would be given by $\hat{d}_j^{(i+1)} = \text{sign}(\text{Re}(Z_j^{(i+1)}))$ for $j \in U(i+1)$.)

There are many special cases of the preceding framework. For example, $U(i) \sim \{1, ...K\}$ corresponds to *parallel interference cancellation,* wherein each data symbol is simultaneously updated at every stage [28, 4]. *Serial interference cancellation (SIC)* corresponds to updating one data symbol at a time. An example of a fixed serial update schedule might be $U(i) = i \bmod K + 1$, where the data symbols are updated one at a time in a fixed cyclic order [21]. It is, of course, possible to devise interference cancellation schemes that are neither parallel nor serial, and to choose the update schedule based on the receiver statistics or side information regarding the signal vectors.

Interference cancellation methods have much potential as a class of low-complexity schemes for attempting to approach ML performance. While our theoretical understanding of these schemes is somewhat limited, the large body of literature on these schemes does reveal the following trends:
1) The performance of is sensitive to the initial estimates $\hat{\mathbf{d}}^{(0)}$. For example, initialization using decorrelator-based estimates typically performs much better than initialization with estimates from matched filter receivers.
2) Parallel interference cancellation updates may exhibit limit cycles [29], while serial interference cancellation updates converge to a local maximum of the

likelihood function [21]. On the other hand, the delay required for updating the data is larger with serial schemes.
3) The performance is better when the signal vectors have disparate energies. In this scenario, demodulating the strongest user and cancelling its contribution before proceeding to the weaker users can yield near-optimum performance, especially for the weaker users [5, 6].
4) Soft decision feedback may be helpful in certain scenarios [4, 21], but has pitfalls for parallel interference cancellation unless properly optimized [2].

Improving our understanding of interference cancellation strategies is an important open problem, as discussed in Section 5. A new interference cancellation technique is described in Section 4.

3.2. Adaptive receivers

Consider first the model (1), assuming that the signal vectors in $\mathbf{U}[n]$ are independent of n. This corresponds to assuming that the channels are time-invariant, and, as described in Section , applies either to a symbol level model for DS-CDMA with short spreading waveforms and narrowband systems, or to a chip level model for DS-CDMA with long spreading waveforms. In this case, the received vector sequence $\{\mathbf{r}[n]\}$ is stationary, assuming that the data sequence $\mathbf{d}[n]$ is stationary. The statistics of the received vector can therefore be learnt, and employed for interference suppression.

The simplest receiver based on the above premise is the linear MMSE receiver, where the correlator $\mathbf{c}_j$ for demodulating d_j is chosen to minimize the mean squared error $E[|\langle \mathbf{c}_j, \mathbf{r}[n]\rangle - d_j[n]|^2]$. The unique solution to this quadratic optimization problem is given by

$$\mathbf{c}_j = \{E[\mathbf{r}[n](\mathbf{r}[n])^H]\}^{-1} E[d_j[n]^* \mathbf{r}[n] \qquad (7)$$

which is independent of n if the joint second order statistics of $\{\mathbf{r}[n]\}$ and $\mathbf{d}[n]$ are independent of n. The standard adaptive implementations of such receivers may be interpreted as arising from replacement of the statistical averages involved in the cost function and solution by empirical averages over n. Note that computation of such an average in (7) requires a training sequence for the desired data $\{d_j[n]\}$. Note that no explicit information about the interference is required. The linear MMSE receiver may be interpreted as a whitened matched filter which deemphasizes the directions in which the interference is strong, and is therefore immune to the near-far problem.

The requirement for a training sequence can be removed by insisting instead on knowledge of the desired signal vector $\mathbf{u}_j$. In this case, the MMSE receiver can be reformulated as a constrained minimum output energy (CMOE) receiver [10], which minimizes the output energy $E[|\langle \mathbf{c}_j, \mathbf{r}[n]\rangle|^2]$, subject to the constraint that $\langle \mathbf{c}_j, \mathbf{u}_j\rangle = 1$. The constraint fixes the desired signal vector's contribution to the output, so that minimization of the output energy becomes equivalent to minimization of the interference energy. The resulting (semi-)blind adaptive algorithm, however, has the drawback of having a higher misadjustment at steady state [10, 22], and is vulnerable to *mismatch* regarding the estimate of the desired signal vector. The problem of mismatch, which can cause suppression of the desired signal, can be alleviated by imposing a variety of further constraints.

For multiuser systems in which the number of users is significantly smaller than the number of dimensions, the use of subspace methods can improve both complexity and performance [33]. Another means of reducing the complexity is the multi-stage Wiener filter [8, 11].

Joint synchronization and interference suppression: The preceding formulation assumes that the receiver is synchronized to the user of interest. For example, for a training based MMSE receiver, the receiver must know which element of the training sequence is in the current observation interval. If, further, the Doppler shift of the desired user due to the channel is unknown, then the received training symbols are actually multiplied by a complex scale factor which rotates at the rate of the Doppler. For a CMOE receiver, estimation of the desired signal vector requires knowledge of the propagation channel as well as of the spreading waveform for the desired user. The following basic approach has been shown to work well in simple special cases [16, 17]: the uncertainty regarding the unknown quantities is quantized into a discrete set of hypotheses, adaptive receivers are run under each hypothesis, and a choice is made among the set of receivers thus obtained based on certain empirically computed statistics. For a multipath channel where the signal vectors are of the form ((2), with slowly varying h_{jl} and constant $\mathbf{s}_{jl}$, the CMOE approach needs to be implemented with additional constraints in order to avoid the problem of mismatch [25, 26, 17].

Channel time variations: Consider now the simplest perturbation of the assumption of channel time invariance, a single path fading channel, where the signal vectors take the form $\mathbf{u}_j[n] = h_j[n]\mathbf{s}_j$. Recall from (7) that the linear MMSE solution requires computation of $E[d_j^*[n]\mathbf{r}[n]]$. Under standard assumptions, this reduces to equals $E[|d_j[n]|^2]E[h_j[n]]\mathbf{s}_j$. Thus, if the expected value of the channel gain $E[h_j[n]]$ (or rather, its empirical average based on the time constant of the adaptive algorithm) is close to zero, then so is the linear MMSE correlator. Thus, the linear MMSE criterion cannot track a channel that varies quickly relative to the time constant of the adaptive algorithm. In fact, for typical data rates and vehicular speeds for cellular telephony, standard linear MMSE adaptation does break down because of the preceding reason. The CMOE receiver of [10] is robust to fading, but its higher misadjustment and sensitivity to mismatch are significant drawbacks.

One of the solutions to LMMSE adaptation over fading channels, versions of which have been proposed by several authors [1, 12, 13, 3], is to track the channel of the desired user by some other means, and to include an estimate of the channel in the cost function. Thus, the receiver for d_j would minimize $E[|\langle \mathbf{c}_j, \mathbf{r}[n]\rangle - h_j[n]d_j[n]|^2]$, which has the following solution

$$\begin{aligned} \mathbf{c}_j &= \{E[\mathbf{r}[n](\mathbf{r}[n])^H]\}^{-1}E[d_j[n]^* h_j^*[n]\mathbf{r}[n]] \\ &= \{E[\mathbf{r}[n](\mathbf{r}[n])^H]\}^{-1}E[|d_j[n]|^2]E[|h_j[n]|^2]\mathbf{s}_j \end{aligned} \quad (8)$$

Note that this solution involves an average of the (positive) energy of the channel gain h_j, rather than of the (typically zero mean) gain itself. Thus, insertion of a prior estimate of the channel succeeds in relieving the adaptive algorithm of the burden of tracking the channel.

The Differential MMSE (DMMSE) criterion: Another solution to channel time variations is to modify the cost function to exploit the relatively slow variation of the channel as a function of n. Thus, if $\mathbf{c}_j$ is effective in suppressing interference, then $\langle \mathbf{c}_j, \mathbf{r}[n]\rangle$ and $\langle \mathbf{c}_j, \mathbf{r}[n-1]\rangle$ should differ roughly by a factor $\frac{d_j[n]}{d_j[n-1]}$. This intuition leads to the differential MMSE (DMMSE) cost function [18], where we minimize

$$E[|\langle \mathbf{c}_j, \mathbf{r}[n]\rangle - \frac{d_j[n]}{d_j[n-1]}\langle \mathbf{c}_j, \mathbf{r}[n-1]\rangle|^2] \quad (9)$$

subject to

$$E[|\langle \mathbf{c}, \mathbf{r}[n]\rangle|^2] = 1 \quad (10)$$

The constraint (10) eliminates the zero correlator as the minimizer for the cost function (9), and turns out to be precisely what is needed to make the solution to the DMMSE problem proportional, under standard assumptions, to the linear MMSE correlator. The DMMSE criterion avoids tracking the channel of the desired user altogether, which means that the desired symbol sequence is recovered only up to a complex scale factor. This ambiguity is assumed to be resolved in other ways, such as by the use of differential modulation or pilot symbols. Adaptive algorithms for obtaining the DMMSE correlator, analogous to those for the linear MMSE criterion, are available, and provide robust performance over fading channels. These adaptive algorithms extend naturally [37] to multipath fading channels of the form (2).

DMMSE for long spreading waveforms: For the chip level model for CDMA with long spreading sequences, $d_j[n]$ equals the product of the (unknown) symbol and the (known) spreading sequence. Since each symbol remains constant over several chips of the spreading sequence, within a symbol, the ratio $\frac{d_j[n]}{d_j[n-1]}$ simply equals the ratio of two successive chips of the spreading sequence of the desired user, which is known by the receiver. Thus, the receiver has a *perpetual chip-rate training sequence* for implementing the DMMSE algorithm for chip level interference suppression, while being "blind," in that no training regarding the desired symbol sequence is required! Simulations of receive antenna array adaptation in such a scenario [18] show that the DMMSE-based algorithm converges significantly faster than any other available blind algorithm for DS-CDMA with long spreading waveforms.

3.3. The Role of Coding

In principle, a coded CDMA system can achieve multiuser channel capacity over an AWGN channel, where each user employs codes for the singleuser AWGN channel, and where the receiver uses successive interference cancellation, or stripping, *after decoding* [23]. Note that this differs from the interference cancellation strategies discussed previously, which operated on the symbols *prior to decoding.* Even assuming that the ideal channel estimates assumed to be available for implementation of interference cancellation are available, a disadvantage of post-decoding cancellation is that the overall decoding delay, which is proportional to the number of users, does not scale well in a large system, especially since the delay involved in decoding a code operating close to channel capacity is large. For a relatively small number of users, a parallel form of such joint multiuser detection and decoding can also be obtained, a particularly powerful combination of which is through turbo decoders exchanging information with linear MMSE detectors (the latter are modified to account for the soft data feedback obtained from the turbo decoders) [34].

The capacity of multiuser systems when the multiuser detection algorithm is used as a front end, followed by separate decoding for each user, has been characterized in a number of scenarios, exploiting large system asymptotics in which the system dimension and the number of users gets large [31]. Large system asymptotics have also been employed to characterize the performance of uncoded systems employing linear MMSE reception [27].

4. PASIC: A NEW INTERFERENCE CANCELLATION TECHNIQUE

Serial interference cancellation (SIC) has the desirable property of converging to a local maximum of the likelihood function, or equivalently as in (4), a local minimum of the distance $||\mathbf{r} - \mathbf{U}\mathbf{d}||$. The idea of our Multithread Arbitrated Serial Interference Cancellation (PASIC) algorithm is quite simple:

(a) Run multiple SIC algorithms in parallel, each with a different order of users being updated, to arrive at different candidate data sequences, each corresponding to a local maximum of the likelihood function (in our numerical results, we run the SIC algorithms for $2K$ updates rather than waiting for convergence).

(b) Choose the candidate that is the most likely, i.e., that minimizes $||\mathbf{r} - \mathbf{U}\mathbf{d}||$.

In our simulations, we consider a DS-CDMA system with processing gain $N = 20$, and with random binary spreading sequences. Thus, the entries of the signal matrix $\mathbf{U}$ are independent and identically distributed (iid) random variables taking values in $\{-1, 1\}$ with equal probability. We first consider equal powers, with E_b/N_0 for each user set at 10 dB (this may be interpreted as the signal-to-noise ratio, including the unmodeled inter-cell interference in the noise power). The receiver knows $\mathbf{U}$, and obtains the matched filter sufficient statistics $\mathbf{Z}$ as in (3). Binary antipodal signaling is used, with the entries of the data vector $\mathbf{d}$ modeled as iid, taking values in $\{-1, 1\}$ with equal probability. The matched filter data estimates $\hat{\mathbf{d}}_{MF} = \text{sign}(\mathbf{Z})$ are employed as the initial conditions for SIC. The PASIC algorithm employs four SIC algorithms in parallel, each with a different update order. Its performance is compared to that of the matched filter receiver and of a single SIC algorithm (we simply use one of the parallel algorithms in PASIC, all of which are equivalent in the equal power case, for this purpose). Each SIC algorithm in PASIC uses $2K$ iterations, so that each user's bit is updated twice.

It is worth noting that, in the range $10^{-2} - 10^{-1}$, which is the typical uncoded error probability required to obtain good coded performance with powerful low-rate convolutional or turbo codes, the PASIC algorithm provides significant capacity gains over both the matched filter receiver and the SIC algorithm. In particular, using an uncoded bit error probability of 10% as our target (this is a "magic number" at which classical turbo codes are extremely effective), the number of users that can be supported is about $1.5N$ using the

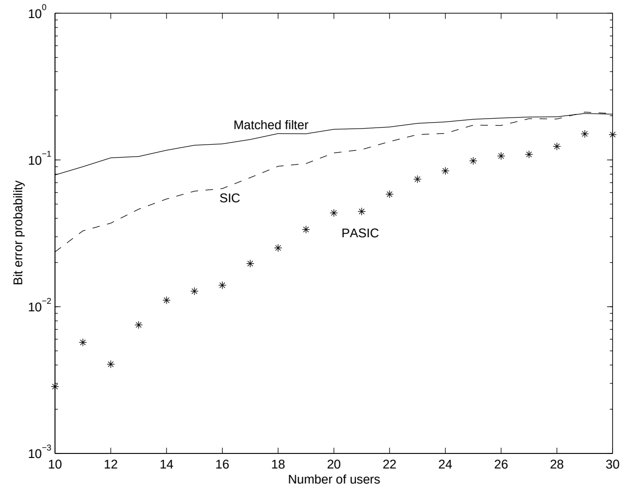

Figure 1: Bit error probability (averaged over runs and users) versus number of users when all users have equal powers.

PASIC detector, about N using SIC, and about $.5N$ using the matched filter.

Figure 2 illustrates the immunity of the PASIC and SIC detectors to the near-far problem, displaying the error probability for the weak users when half ($\lceil K/2 \rceil$) of the users are 20 dB stronger than the other half, with $E_b/N_0 = 10$ dB for the weak users. The performance of the matched filter is predictably poor.

5. SOME OPEN PROBLEMS

Third generation cellular standards will probably employ DS-CDMA with long spreading waveforms. Low-complexity multiuser detection methods that are well-matched to such systems include linear adaptive MMSE and DMMSE interference suppression at the chip level, interference cancellation techniques at the symbol level, and combinations of these two classes of multiuser detection schemes. From a practical perspective, a general issue that needs further thought is the design of multiuser detection algorithms that provide good performance in conjunction with powerful error control codes. In such a setting, the uncoded bit error probabilities are quite high, so that the traditional asymptotic comparisons at high signal-to-noise ratios have limited significance. One desirable property that still must be maintained, however, is robustness against the near-far problem. A related theme is the reduction of the complexity and delay for joint multiuser detection and decoding, which is known to be capable of attaining performance close to single-user performance. An important open problem in this context is to gain a deeper theoretical understanding of interference cancel-

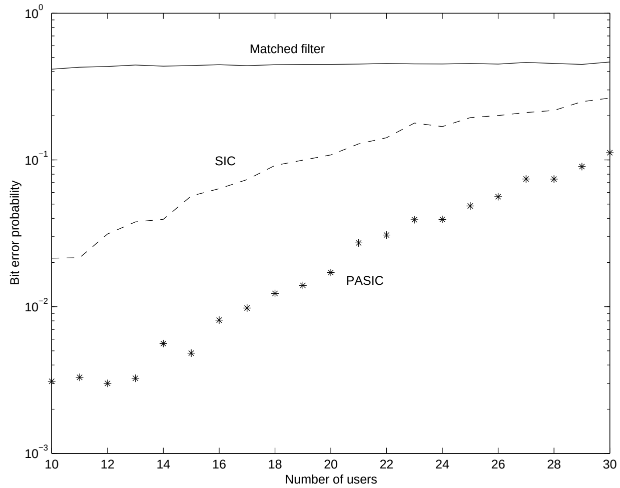

Figure 2: Bit error probability (averaged over runs and weak users) for the weak users, when half of the users are 20 dB stronger than the other half.

lation strategies, in order to design schemes with a level of guaranteed performance, preferably close to that of the ML receiver. Since complexity is a driving concern, interference cancellation schemes that can start with a matched filter initialization are the most desirable. The PASIC strategy described in the previous section is a preliminary step in this direction.

The PASIC detector is analogous to the recently invented BAD (bidirectional decision feedback equalization) algorithm [20] for channel equalization in singleuser systems. The two algorithms have in common the general notion of arbitrating between a set of candidate data sequences based on the match between the received signal and the noiseless reconstructed signals corresponding to the data sequences. It is our hope that this idea will provide a means of approaching ML performance with reasonable complexity in a variety of settings, with the main creative challenge in each scenario being the generation of a small set of good candidate data sequences with low complexity.

DS-CDMA with short spreading waveforms and adaptive interference suppression is well suited to bursty, packetized transmission, where the closed loop power control required for matched filter reception may not be possible. Potential examples of such systems are wireless *ad hoc* networks, which are projected to be a major component of future battlefield communications. However, the use of short spreading waveforms in a military context is controversial, since an eavesdropper could potentially exploit the symbol-level cyclostationarity of the system using blind source separation techniques. This can be mitigated by varying the spreading waveform from packet to packet. The issue then is: what is the largest packet size that can be employed (thus maximizing the ratio of the payload to the overhead for training and other purposes), while not permitting a blind source separator to reliably demodulate the data in the packet? The community attending this workshop is perhaps the best equipped to answer such a question.

An important practical issue is the integration of multiuser detection into synchronization. While a number of approaches to this problem appear in the literature (see [16, 17] and the references therein), much work remains in devising low-complexity approaches to this problem for highly loaded systems.

Multiuser detection is also relevant to the efficient decoding of space-time codes [24, 9, 35], which are designed to provide transmit antenna diversity for combating fading on wireless channels. For a single receive antenna element (which is typical in the base-to-mobile downlink of a cellular system), the signals sent from different transmit elements interfere with each other at the receiver. The decoding of space-time codes can therefore be viewed as a multiuser detection problem with the special property that the different users are sending correlated information. The translation of this connection into efficient decoding algorithms for space-time codes, especially when the channel gains from the transmit antenna elements to the receiver are unknown, is a topic of current investigation. This problem is related to the general topic of multiuser detection for CDMA with nonlinear modulation [32].

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