A SYSTEM-THEORETIC FOUNDATION FOR BLIND SIGNAL SEPARATION OF MIMO-FIR CONVOLUTIVE MIXTURES – A REVIEW

Ruey-wen Liu[1], Yujiro Inouye[2] and Hui Luo[3]

Abstract

The blind demixer (blind signal separation) of a convolutive mixture is investigated. We first present the *characterization* of *reversible mixers* (systems), the broadest class for the purpose of demixing. We then presented the existence of a fundamental factorization property, called the *irreducible-paraunitary factorization*. Based on these fact, we show the limitation of second-order statistics (SOS) for blind demixing when the source signals are white. We then show a sufficient condition for blind demixing by use of higher order statistics. We also present a sufficient condition by use of temporal SOS when the signals are not white

Index Terms: Blind signal separation, Blind deconvolution, MIMO-FIR systems, Independent Component Analysis. Higher order statistics.

I. Introduction

Blind Signal Separation (BSS) was first proposed by Jutten and Herault [1], and later Principle Component Analysis (ICA) was initiated by Comon [2]. A good review of these subject is given by Cardoso [3], together with extensive references. The investigation of these subject at early stage was concentrated on constant mixers. It then evolved to convolutive mixer but with signal source signal. See a good review paper on this subject by L. Tong [4]. Some recent papers [5] – [9] considered the case of MIMO convolutive mixers, but they all considered a class of mixers, called the irreducible mixers. However, irreducible mixers are too restrictive for many applications. It implies that source signals transmitted by different users have to be received *simultaneously* by an antenna array. This requirement is very restrictive in mobile communications, where source signals may reach the antenna array with different delays.

We considered the class of FIR convolutive mixers (systems), called reversible mixers, the broadest class for the purpose of demixing. The technique used for irreducible mixers are heavily based on a system theoretical foundation on irreducible systems by Forney [10]. Since no such foundation for reversible mixers can be found in the literature, these techniques can not be directly extended to the case of reversible mixers.

In this paper, we first present a characterization of reversible mixers. An important factorization theorem, called the irreducible-paraunitary factorization, is then presented.

Based on this decomposition, we presented a limitation of the second order statistic for blind demixing, which states that the best SOS can do is to blindly deconvolve the mixers. We also give the set of reversible mixers in an explicit form that can be blindly deconvolved by use only second order statistics. This set includes the irreducible channels as presented by other authors. We also present two (very tight) sufficient conditions for blind demixing,

[1] University of Notre Dame, Notre Dame, IN 46556, USA. Email: liu.1@nd.edu
[2] Shimane University, 1060 Nishi-Kawatsu, Matsue Shimane 690-8504, Japan.
Email: inouye@riko.shimane-u.ac.jp
[3] AT&T Research Lab, Red Bank, NJ 07701, USA. Email: luo@research.att.com

one using higher order statistics, and one using SOS temporal statistics.

II. Preliminaries

The concepts to be presented below have been used by many authors with different names. We try to investigate this subject in a formal way.

Let $C[z]^{m\times n}$ denote the set of $m\times n$ polynomial matrices in *z* with coefficients in C. We are particularly interested in the case when *z* is the delayed operator, i.e., $zx(t)=x(t-1)$, and in this case, each entry represents a finite impulse response (FIR) filter. We also denote $\Lambda(z)$ a diagonal matrix with monic monomial diagonal elements, i.e., z^{l} for some $l\geq 0$. Finally, we denote A^H, A^T *and* A^* the Hermitian, the transpose, the complex-conjugate of *A* respectively.

A square matrix $G(z)\in C[z]^{n\times n}$ is said to be *transparent* if it has a decomposition of the form:

$$G(z) = PD\Lambda(z), \qquad (1)$$

where *P* is a permutation matrix and *D* a regular diagonal constant matrix. When two signals *z(t)* and *s(t)* are related by a transparent matrix *G(z),*

$$y(t) = G(z)s(t), \qquad (2)$$

they are essentially the same except for a permutation ambiguity, an amplitude ambiguity, and a delay ambiguity. Hence, *transparency is a weaker form of identity*.

A matrix $H(z)\in C[z]^{m\times n}$ is said to be *reversible* if there exists $W(z)\in C[z]^{m\times n}$ such that the product

$$G(z) := W^T(z)H(z) \qquad (3)$$

is transparent, i.e., satisfies (1). Hence *reversibility is a weaker form of invertibility*. The subject of invertible matrices has been well studies in the literature, but much less so for reversible matrices.

The above discussion is related to a mixer-filter combination. Let *x(t)* and *s(t)* be related by

$$x(t) = H(z)s(t) \qquad (4)$$

where s(t) is the *source signal*, $H(z)\in C[z]^{m\times n}$ the *mixer (or mixing matrix)*, and x(t) the *mixer output*. The mixer output *y(t)* is a *constant* or *convolutive* mixture of *s(t)* if *H(z)* is respectively a constant or a non-constant polynomial matrix. To process *x(t),* we cascade it with $W(z)\in C[z]^{m\times n}$, called a *filter*. Its input/output relation is described by

$$y(t) = W^T(z)x(t) \qquad (5)$$

where *y(t)* is the *filter output*. The overall response of the mixer-filter combination is described by

$$y(t) = G(z)s(t) \qquad (6)$$

where $G(z) := W^T(z)H(z)$ is the *total system*.

In relation to a mixer $H(z)$ we want to design a filter $W(z)$ so that total system $G(z)$ achieves certain objective. The *objective for blind demixing* is for *G(z)* to be transparent, and the filter *W(z)* that achieved it is called the *demixer*. *Demixing is a weaker form of inversion*. The adjective *blind* stresses the fact that this is accomplished with no or little knowledge of the mixing matrix *H(z)* and the signal *s(t).*

The terminology chosen here are application-independent. Other terminology has been used for specific applications. For example, “reversible” has been called “*equalizable*” for communication applications. Demixing is often called *signal separation* in signal processing applications.

III. A System Theoretic Study of Demixing of Convolutive Mixtures

Let us first investigate the characterization of reversible matrices. Another concept called irreducible matrices that is well investigated

[10], [11] and well applied [5] – [9]. Since the characterization of these two kinds of matrices are very similar, it is instructive to put them together in one theorem,

Theorem 1 (Characterization of reversible and irreducible matrices): A necessary and sufficient condition for a matrix $H(z) \in C[z]^{m\times n}$ to be reversible (or, irreducible) if and only if one of the following equivalent conditions is satisfied:

a) The rank $H(\lambda) = n$ for all nonzero $\lambda \in C$ (or, for all $\lambda \in C$)
b) The Smith form of *H(z)* has the form:

$$U(z)H(z)V(z) = \begin{bmatrix} \Lambda(z) \\ 0 \end{bmatrix}, \text{ (or, } = \begin{bmatrix} I \\ 0 \end{bmatrix})$$

   where *U(z)* and *V(z)* are unimodular matrices.
c) There exists a reversible (or, irreducible) $W(z) \in C[z]^{m\times n}$ such that $W^T(z)H(z) = z^{l} I$ for some $l \geq 0$ (or, for $l = 0$).
d) The GCD of all the $n \times n$ minors of $H(z)$ is z^{l} for some $l \geq 0$ (or, $l = 0$).

The part of the Theorem on irreducible mixers can be found in Forney [10] or Kailath [11]. The part on reversible mixers can be found scattered places, also can be easily derived.

Comparing Condition (a), clearly irreducible matrices is a subset of reversible matrices. Although these two sets of matrices are very close in their mathematical descriptions, they are very different in their domain of applications. Note that Condition (a) implies that *H(0)* is of full column rank for irreducible matrices. Since *H(0)* is also the leading coefficient matrix of *H(z)*, it implies that the signals from all sources have to arrive at an antenna array (the receivers) at the same time. This condition is too restrictive for some communication applications. On the other hand, *H(0)* need not be of full column-rank for reversible matrices and they can admit the case when signals arrive receivers randomly. For this reason, it is important to investigate the case of reversible matrices.

The relation between reversible matrices and irreducible matrices is shown in the next theorem. Let us first recall that a square polynomial matrix $H_P(z) \in C[z]^{n\times n}$ is said to be *paraunitary* if it is unitary on the unit cycle, *i.e.*, $H_P(e^{-j\omega})H_P^H(e^{-j\omega}) = I$ for all ω; and it is said to be *unimodular* if its determinant is a nonzero constant.

Theorem 2 (Irreducible-Paraunitary Factorization) [18], [19]*:* Any reversible polynomial matrix $H(z) \in C[z]^{m\times n}$ can be factorized as follows:

$$H(z) = H_I(z)H_P(z), \qquad (7)$$

where $H_I(z)$ is irreducible and $H_p(z)$ is paraunitary. Moreover, the above factorization is unique up to the multiplication of a unitary matrix $U \in C^{n\times n}$.

The fact that *U* is constant is nontrivial.

Theorem 2 provides a system theoretical foundation for the investigation of the mixing matrices that are reversible. In communication, such mixing matrices have applications including, for example, Rayleigh-fading channels in a mobile communication system in which the signals arrive randomly with delays together with unknown channel orders.

Finally, the order of the demixer is investigated. Let the order of the mixing matrix *H(z)* be bounded by L_H. It can be shown that there exists a demixer $W(z)$ with order less or equal to $L_W = 2nL_H - 1$, where *n* is the number of sources [18].

IV. Conditions and Limitations for Blind Demixing

In this section, we first show in Theorem 3 that SOS is not enough for the demixing of a convolutive mixture. The best it can do is deconvolve the mixer. The exact set of reversible mixers that can be deconvolved blindly by SOS is presented. A (very tight) sufficient condition for blind demixing by use of higher order statistics is given in Theorem 4. Another (very tight) sufficient condition for blind demixing by use of SOS Temporal statistics is given in Theorem 5. First we will show that why the SOS techniques used for irreducible mixer failed for reversible mixers.

The case for irreducible (or, reversible) mixers $H(z) \in C[z]^{m \times n}$ can be summarized by a Bezout-type Identity as follows.

$$\begin{bmatrix} W^T(z) \\ W_B{}^T(z) \end{bmatrix} \begin{bmatrix} H(z) & H_C(z) \end{bmatrix} = z^{l} I \quad (8)$$

for $l = 0$ (for some $l \geq 0$). In this identity, several system theoretical results are used. First, given an irreducible (or, reversible) $H(z) \in C[z]^{m \times n}$, there exists an irreducible (or, reversible) $H_C(z) \in C[z]^{m \times (m-n)}$ such that the square matrix $\begin{bmatrix} H(z) & H_C(z) \end{bmatrix} \in C[z]^{m \times m}$ is unimodular (or, reversible). As such, there exists an unimodular (or, reversible) $\begin{bmatrix} W^T(z) \\ W_B{}^T(z) \end{bmatrix} \in C[z]^{m \times m}$ such that the Bezout Identity is satisfied. This matrix equation implies that $W(z)$ is a demixer and $W_B(z)$ called a *blocker* for $H(z)$, because respectively, they satisfy $W^T(z)H(z) = z^{l}\mathrm{I}$ and $W_B{}^T(z)H(z) = 0$. The key step for blind technique shows that a blocker $W_B(z)$ can be obtained blindly by second order statistics, such as by a subspace method [5,6]. Since the space spanned by the columns of $H(z)$ is the orthogonal complement of that of $W_B(z)$, it can be uniquely identified once $W_B(z)$ is obtained. The final step depends heavily on a system theoretic theorem by Forney [10] that when *H(z)* is irreducible and column-reduced, its range space uniquely determines its columns up to a constant matrix. It concludes that *H(z)* can be blindly identified up to a constant matrix by SOS techniques, if it is irreducible and column reduced. However, since there is no similar result for reversible mixers, this blind technique can no longer be applied to $H(z)$ that is reversible but not irreducible.

A different blind technique will be introduced here. Instead of identify $H(z)$ first and then find a demixer for it, we propose to approach the problem directly, i.e., study the total system *G(z)* directly. First, define a filter $W(z) \in C[z]^{m \times n}$, called a *deconvolver*, if it satisfies

$$G(z) := W^T(z)H(z) = C\Lambda(z) \quad (9)$$

where C is a regular constant matrix. A deconvolver eliminates the dynamically mixed effect of the mixer $H(z)$, but it is weaker than a demixer. The following theorem is a modification of a result given in [18], [19].

Theorem 3: Let $H(z) \in C[z]^{m \times n}$ be reversible and let the source signal $\{s(t)\}$ be zero-mean and its SOS satisfy the following,

$$E\{s(t+k)s^H(t)\} = I\delta(k), \ \forall t \quad (10)$$

where $\delta(k)$ is the Kronecker delta, i.e., it is normalized and uncorrelated both temporally and spatially. Then, it can be deconvolved by SOS if and only if the mixer has the form

$$H(z) = H_I(z)U\Lambda(z). \quad (11)$$

The proof is essentially given in [19]. It is given here to show that the factorization theorem (Theorem 2) is critical for the investigation.

Proof: Since the input signal is uncorrelated both temporally and spatially, it is necessary that the output *y(t)* of the filter is whitened in order to deconvolve $H(z)$. As such, the total system has the form

$$G(z) := W^T(z)H(z) = D\tilde{H}_P(z) \quad (12)$$

where D is a regular diagonal constant matrix, and $\tilde{H}(p) \in C[z]^{n \times n}$ is paraunitary. This can be achieved by using SOS technique such as a linear predictor, and in which the filter *W(z)* is irreducible [18], [19]. This equation represents all the information can be obtained by SOS, under the assumption of (8) and with the objective of deconvolving *H(z)*. Using the irreducible filter $W_I(z)$ and the decomposition $H(z) = H_I(z)H_P(z)$, the above equation becomes

$$W_I^T(z)H_I(z)H_P(z) = D\tilde{H}_P(z) \qquad (13)$$

Since both sides of it are an irreducible-paraunitary decomposition, by the uniqueness theorem of theorem 2, there exists a unitary matrix $\hat{U}$, such that

$$W_I^T(z)H_I(z) = D\hat{U}. \qquad (14)$$

and

$$H_P(z) = \hat{U}^H\tilde{H}_P(z) \qquad (15)$$

Now, suppose that (11) is satisfied. Substitute it to (14), we have

$$W_I^T(z)H(z) = D\hat{U}U\Lambda(z)$$

which has the form of (9). Hence, the system is deconvolved. On the other hand, let it be deconvolved. Comparing (9) and (12), we have $D\tilde{H}_P(z) = C\Lambda(z)$. Substitute it into (15), we have

$$H_P(z) = \hat{U}^H\tilde{H}(z) = \hat{U}^H D^{-1}C\Lambda(z)$$

where $(\hat{U}^H D^{-1}C)$ is unitary. The theorem is proved.

This shows the limitation of SOS. When the input sequence is white, the best second-order statistics can do is to deconvolve the mixer, and this can be done only to a special class of reversible mixers. One must us higher order statistics for the demixing. It is surprising that we do not need to many higher order statistics in order to demixing the reversible mixers.

Theorem 4 [14], [15]: Under the hypotheses of theorem 3, if in addition the source signals *s(t)* are spatially fourth-order white, then *G(z)* is transparent if and only if *y(t)* satisfies the same condition as that for *s(t)*, i.e., temporally white and spatially second- and fourth-order uncorrelated.

Both theorem 3 and 4 concern the case when temporal statistics of the signals s(t) are zero, i.e., white. When the temporal statistics of the signals are non-zero, blind demixing actually can be achieved by using only SOS, to be presented as follows.

Recall the total system described by (6),

$$y(t) = G(z)s(t)$$

Denote the correlation functions by $r_i(\tau) = E\{s_i(t)s_i^*(t-\tau)\}$ and $\rho_i(\tau) = E\{y_i(t)y_i^*(t-\tau)\}$. Let L denote a bound of the order of *G(z)*. Let Ω denote the set of autocorrelation functions of the source signal *s(t)*.

$$\Omega = \{r_i(\tau-k) \mid i=1,\ldots,n;\ k=0,\pm1,\ldots,\pm L\}$$

Theorem 5 [20]: Let $G(z) \in C[z]^{n\times n}$. Let the set of autocorrelation functions in Ω be linearly independent. Then for each $i = 1,2,\ldots,n$,

$$y_1(t) = d_i z^{l_i} s_i(t)$$

for some d_i and $l_i \geq 0$ if and only if $\rho_1(\tau) = r_i(\tau)$.

Corollary 5.1: Under the hypotheses of Theorem 5, *G(z)* is transparent if and only if the autocorrelation functions of *y(t)* is the same as any permutation of that of *s(t)*.

Theorem 5 has the implication of a solution of multi-user problems. An implementation of the theorem is given in [20].

V. Conclusion

We considered the class of FIR mixers called reversible mixers, the broadest class for the purpose of demixing. A characterization of reversible mixers is first given. We then show that any reversible mixer can be written as a product of an irreducible mixer and a paraunitary mixer. Based on this fact, we then show a limitation of SOS. Specifically, SOS does not have enough information to do blind demixing when the source signal is white. The best it can do is to deconvolve a class of reversible systems. The explicit form of this class of reversible systems is given in Theorem 3. We then show in Theorem 4 that the SOS together with the fourth order spatial statistics of the signal do have sufficient information for blind demixing. Finally, we consider the case when signal is not while. It is shown in Theorem 5 that if the set of autocorrelation functions of the signals together with their displacements is linearly independent, then the second-order temporal statistics do have sufficient information for blind demixing.

Noise $n(t)$ is ignored in this paper for the purpose of analysis. This can be justified if its second order statistics can be removed, for example by a subspace method.

Finally, since this is a review paper, all proofs but the one for Theorem 3 are omitted but referenced.

REFERENCES

[1] J. Herault, C. Jutten, and B. Ans, “Detection de grandeurs primitives dans un message composite par une architecture de calcul neuromimetique en apprentissage non supervise,” in *Proc. GRETSI*, Nice, France, pp. 1017-1020, 1985.

[2] P. Comon, “Independent component analysis, a new concept?”, *Signal Processing*, pp. 287-314, Apr. 1994.

[3] J. -F. Cardoso, “Blind signal separation: statistical principles,” Special Issue on Blind System Identification and Estimation, Proc. of IEEE, pp. 2009-2025, October 1998.

[4] L. Tong and S. Perreau, “Multichannel blind Identification: From subspace to maximum likelihood methods,” Special Issue on Blind system identification and estimation, *Proceedings of IEEE*, pp. 1951-1968, October 1998

[5] D. Slock, “Blind joint equalization of multiple synchronous mobile users using oversampling and/or multiple antennas,” Proc. Asilomar Conf., October 1994.

[6] D. T. Slock, "Blind fractionally-spaced equalization, perfect reconstruction filter-banks and multichannel linear prediction," in *Proc. ICASSP*, vol. 4, pp. 585-588, 1994.

[7] K. Abed Merain, P. Loubaton, and E. Moulines, "A subspace algorithm for certain blind identification problem," *IEEE Trans. Inform. Theory*, vol. 43, pp. 499-511, March 1997.

[8] A. Gorokhov and P. Loubaton, "Subspace-based techniques for blind separation of convolutive mixtures with temporally correlated source," *IEEE Trans. Circuits Syst.*-I, vol. 44, pp. 813-820, Sept. 1997.

[9] A. Gorokhov and P. Loubaton, "Blind identification of MIMO-FIR system: a generalized linear prediction approach,” *Signal Processing*, 1999.

[10] G. Forney, “Minimum bases of rational vector spaces with applications to multivariable linear systems,” *SIAM J. Contr*., vol. 13, pp. 493-520. May 1975.

[11] T. Kailath, *Linear Systems*. New York: Prentice Hall, 1980.

[12] Special Issue on Signal Processing for Advanced Communications, *IEEE Trans. Signal Processing*, vol. 45, Jan. 1997.

[13] L. Tong, G. Xu, and T. Kailath, "A new approach to blind identification and equalization of multipath channels," in *Proc. 25th Asilomar Conf.*, Pacific Grove, CA, 1991, pp. 856-860.

[14] Y. Inouye and R. Liu, "Criterion for direct blind deconvolution of MIMO FIR systems driven by white source signals," in *Proc. ICASSP*, Phoenix, AZ, 1999, paper #1138.

[15] R. Liu and Y. Inouye, "Direct signal separation of convolutive mixtures of white spatially independent non-Gaussian signals," in *Proc. ICA'99 (1st Int. Workshop on Independent Component Analysis and Signal Separation)*, Aussois, France, pp. 233-238, 1999.

[16] R. Liu and Y. Inouye, "Direct blind deconvolution of multi-user-multichannel systems," in *Proc. ISCAS, Orlando, FL*, vol. 5, pp. 49-52.

[17] J. K. Massey and M. K. Sain, "Inverse of linear sequential circuits," *IEEE Trans. Computers*, pp. 330-337, 1968.

[18] Y. Inouye and R. Liu, “A System-Theoretic Foundation for Blind Equalization of an FIR MIMO Channel System,” *Proc. ISCAS, May 2000*, to appear.

[19]Y. Inouye and R. Liu, “A System-Theoretic Foundation for Blind Equalization of an FIR MIMO Channel Systems,” *Submitted for publication.*

[20] H. Luo and R. Liu, “Second-Order Statistics Blind MIMO-FIR Channel Equalization for Antenna Array-Based Wireless Communications Systems,” *IEEE Sensor Array and Multichannel (SAM) Signal Processing Workshop* , (to appear) Cambridge, Massachusetts, March 2000.