A CATASTROPHE THEORY MODEL FOR THE WORKING-MEMORY OVERLOAD HYPOTHESIS - METHODOLOGICAL ISSUES

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ABSTRACT

The present work is a part of educational research, which explores the applicability of Catastrophe Theory for testing nonlinear hypotheses in this field. This part is an extension of previous investigations and contributes to the understanding the nature of mental processes involved in problem solving. Research has shown that students' achievement in science education and particularly in problem solving is associated with some psychometric /cognitive variables, such as information processing capacity (working memory capacity and/or mental capacity). The information processing however, interferes with other variables, such as cognitive style or logical thinking, which could act as inhibitory processes. That could lead to working memory overload and to students' failure. The present work adds to the previous research by examining the working memory overload hypothesis as a catastrophe effect and proposes a cusp model, which accounts for discontinuities in students' performance. Data were taken from a tenth-grade student examination in chemistry and physics. Measurements were taken at two points in time, and the data analysis involved dynamic difference equations and statistical regression techniques. The model implements two cognitive variables as controls: Baddeley's working-memory capacity as asymmetry factor and logical thinking as bifurcation. It proved to be superior to the linear counterparts, and that shows the limitations of the general linear model (GLM). The model, which is a phenomenological one, demonstrates nonlinear interactions between students' mental resources and mental tasks, and supports the dynamical nature of a problem solving process, where nonlinearity is expected. This work contributes to realization of a paradigm shift for research in social and behavioural science. Moreover, it demonstrates the feasibility of providing empirical evidences for nonlinear processes in education research, and it could build bridges between NDS-theory concepts, psychological or/and pedagogical theories. In addition, some philosophical - ontological and epistemological - questions concerning the utilization of catastrophe theory models in social research.

1. INTRODUCTION

1.1. The Working-Memory Overload Hypothesis

Basic research in science education has shown that the information processing theories and models could be utilized in order to understand and explain students' performance. The above theories focus on the capacity limitation of short-term memory, which has been proved to be of great importance mainly in problem solving. Two theoretical constructs prevail in this research area: the theory of Constructive Operators [1] and the working memory (WM) model [2]. They both account for the limited human channel capacity by implementing the concepts of working-memory capacity and mental capacity respectively. However, there are recent theoretical discussions about a possible unification of the two theories [3] [4] [5]. These mental resources express subjects' information processing capacity and they have correlated with successful performance in chemistry and physics problem solving. Working memory capacity and mental capacity are associated with the mental demand of the problem that is "the maximum number of thought steps and processes which have to be activated by the least able, but ultimately successful candidate in the light of what had been taught" [6]. In science education problem solving, a model that is associated with the working-memory overload hypothesis, states that if a student does not fail for lack of information or recall, he/she is likely to be successful in solving a problem if the problem has a *mental demand*, which is less or equal to the subject's information processing capacity, unless the student has strategies that enable him/her to reduce the mental demand of the problem to become less than his/her processing capacity [6]. Thus, as the problem increases in complexity and if the human channel capacity has a final limit, the decrease of achievement may be rapid after the limit has been reached [7]. The rapid decrease in students' achievement has been described as working memory overload and it has been demonstrated by an inverse *S*-shaped curve, which is the graph of students' performance as a function of the mental demand of a problem. The steepest slope of the graph corresponds to subjects' capacity overload. While the theoretical model could provide explanation of students' performance, the related experimental works have not always supported it [8]. This led to the statement of a number of necessary conditions, which must be fulfilled in order for the model to be valid: the logical structure of the problem must be simple; the problem has to be non-algorithmic; the partial steps must be available in the long-term memory and accessible from it; the students do not employ chunking devices; no 'noise' should be present in the problem statement; it holds for field independent students [9]. [10]. The above restricts the model to a few cases and in addition objections have been raised concerning the 'discontinuity' of this sudden drop [8].

1.2. Nonlinear Models in Science Education

The main body of the aforementioned research was based on linear statistics and the model has actually two disadvantages. First, the sudden changes in students' performance were not modeled properly as discontinuities in a mathematical sense. Second, the linear statistical model includes only one variable: the information processing capacity, while it is recognized that additional variables, such as, the degree of field dependence/ independence and logical thinking have immediate impact on the model. An attempt to demonstrate nonlinear changes in students' performance was the development of a complexity theory model, which implemented concepts, such as *fractal dimension*, *order* and *entropy* [11] [12]. In this work, rank-order sequences of the subjects, according to their scores, were generated, and each score was then replaced by the value of the subjects' information processing capacity. Then the sequences were mapped onto a one-dimensional random walk model and when treated as dynamic symbolic sequences, were found to possess fractal dimension or entropy depending on the complexity of the problem. The subjects' capacity overload affected the rank-order achievement scores by changing its order, that is, changing its entropy. The sudden decrease in students' performance was then observed as *phase transition* in one-dimensional system.

A later study tests the nonlinear dynamical hypothesis in science education problem solving by applying catastrophe theory. Within the neo-Piagetian framework a cusp catastrophe model is proposed, which accounts for discontinuities in students' performance as a function of two controls: the functional M-capacity as asymmetry and the degree of field dependence/ independence as bifurcation. The two controls have a functional relation with two opponent processes, the processing of relevant information and the inhibitory process of disembedding irrelevant information respectively. The cusp catastrophe model proved superior compared to the pre-post linear counterpart. This model was supported by the data and provided the basis for building bridges between NDS-theory concepts and neo-Piagetian theories. This study set a framework for the application of catastrophe theory in education.

2. CATASTROPHE THEORY

2.1. Mathematical aspects

Catastrophe theory [13] concerns the study of equilibrium behaviour of a larger class of mathematical system functions that exhibits discontinuous changes. It relates discontinuous changes in dependent variables as a function of continuous variation of the independent variables (controls). CT models in science involve dissipating systems or potential-minimizing systems. Such models ignore the very large number of internal variables, and they constrain the description of the local observed behaviour by a small number of control parameters [14] [15] [16].

An important aspect of CT is the classification Theorem, which states that all discontinuous changes of events can be modeled by one of seven elementary topological forms [17]. These forms are hierarchical and are described by one to four control parameters depending on the complexity of the behaviour they encompass and describe. The elementary catastrophe models are classified into two groups: The cuspoids and the umbilics. The former have drawn most of the attention in social science applications. They involve one dependent variable and have potential functions in three to six dimensions and response surfaces in two to five dimensions. The potential function is the integral of the response surfaces function. They are namely the fold, cusp, swallowtail and the butterfly model [17]. The potential function for the fold catastrophe is

$$V(y,a) = y^3/3 - ay \qquad (1)$$

Where y is the dependent measure and *a* is the control parameter. Its response surfaces function is defined as the set of points where the equation (2) holds:

$$\delta V(y,a)/\delta y = y^2 - a \qquad (2)$$

The most interesting and most applicable is the cusp catastrophe model. The cusp model applies to a system that has two states of stable equilibrium or two *attractors*. It describes changes between two qualitatively distinct forms for behaviour or states. These states within the context encompassed in an educational system could represent *success* or *failure*.

Changes between the two states is a function of the two controls: asymmetry (*a*) and bifurcation (*b*). *a* and *b* are the independent variables and have a particular function in the process of change and in the underlying mechanism. The relation between behaviour *y* and the controls is given by the equation (3), which corresponds to potential function of the cusp catastrophe:

$$V(y,a,b) = y^4/4 - by^2/2 - ay \qquad (3)$$

While the cusp response surface is a set of points where

$$\delta V\,(y, a, b)\,/\,\delta y \;=\; y^3 - by - a \qquad (4)$$

The cusp surface is three-dimensional and features a two-dimensional manifold (Figure 1).

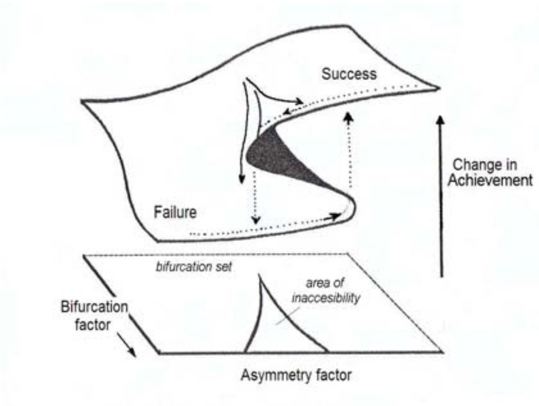

Figure 1. A three-dimensional representation of the cup catastrophe response surface for students' achievement.

For a dynamical system controlled by a potential function the equation dy / dt = - δV / δy holds. Thus, the set of values of y defining the response surface can describe changes in behaviour over time. A description of the cusp response surface is given in section 3.3.

The other catastrophe models are the swallowtail and the butterfly catastrophe, which are described by three and four control parameters respectively [14] [16] [17].

Some important concepts associated with the implementation of CT, especially with the cusp model are the notion of *random force*, which occurs when the system is exposed to an entropy-inducing event (in thermodynamic sense). The random force is applied during time Δt, when the catastrophic event is supposed to occur and it usually tents to increase the entropy of the system. As the *entropy* of the system increases, the probable location of its elements in space expands to the limits of its confinement. Bifurcation can serve as entropy reducing mechanism by partitioning the elements into two areas of relative stability. The elements of the system are not equally exposed to random force because another parameter determines the level of expose and this is the bifurcation parameter in a cusp model [17].

In these nonlinear processes the concept of *causality* differs from the traditional view. In CT model the coaction of random force and bifurcation mechanism are considered jointly and are said to cause the phenomenon, which implies bifurcation, entropy and autonomous process. The concept of cause is replaced by a combination of *control*, and the usual independent variables are referred as *controls*.

The statistical methods which exist and are applied in empirical data are discussed in the last section.

3. EDUCATIONAL RESEARCH

3.1. Methodological aspects for a cusp model

The method presented here involves dynamic difference equations and statistical regression techniques. The dynamic difference equations apply where the behaviour of a system is measured at two points in time. During the time that elapses between the two measurements, time 1 and time 2, it is assumed that a 'random force' has been applied to the system and it is sufficient to excite trajectories across the fullest possible range of the catastrophe manifold and to result a bimodal distribution The bimodality is not necessarily due to the presence of two distinct subpopulations, but it may reflect a nonlinear system having double stable equilibria, that is, two *attractors*. For an educational research during the elapsed time between measurements an intervention or the action of mental resources is assumed. These mental resources are correlated with the variables, which could be implemented as controls in a cusp model. For the present model these are the working-memory capacity (WM) and the logical thinking (L).

For the statistical analysis, the raw scores of the dependent and independent measures were transformed to Z scores corrected for location and scale σ_s.

$$Z = (Y - Y_{\min})/\sigma_s \qquad (5)$$

The location correction is made by setting the zero at the minimum value of Y. The scale, σ_s, represents the variability around the modes or the ordinary standard deviation (σ). The later was used in the present research. The specific equation to be tested for a cusp catastrophe model is:

$$\Delta Z = Z_2 - Z_1 = b_1 Z_1^3 + b_2 Z_1^2 + b_3 L Z_1 + b_4 WM + b_5 \qquad (6)$$

The alternative linear models are the following:

Linear 1 $$\Delta Z = b_1 WM + b_2 L + b_3 \qquad (7)$$

Linear 2 $$\Delta Z = b_1 M + b_2 F + b_3 (WM) L + b_4 \qquad (8)$$

Linear 3 $$Z_2 = b_1 WM + b_2 L + b_3 Z_1 + b_4 \qquad (9)$$

The cusp model holds if equation (6) proved to be superior to the linear alternatives in terms of statistical indices
and variance explained.

3.2. Measurements, Hypotheses and Results

3.2.1. Measurements

In order to apply difference equations, we need to measure achievement at two points in time, Time 1 and Time 2. At Time 1, the Test 1 included knowledge recall questions from related theory, simple calculations (partial steps) and simple conceptual questions. The above assured that a minimum of basic prerequisite knowledge was available in the long-term memory. At Time 2, Test 2 consisted of a demanding problem in physics, which required activation of mental resources such as, WM and logical thinking. Time is implicit here. In addition, all subjects, who were $9^{th,}$ grade secondary school students, were tested for working-memory capacity and logical thinking ability.

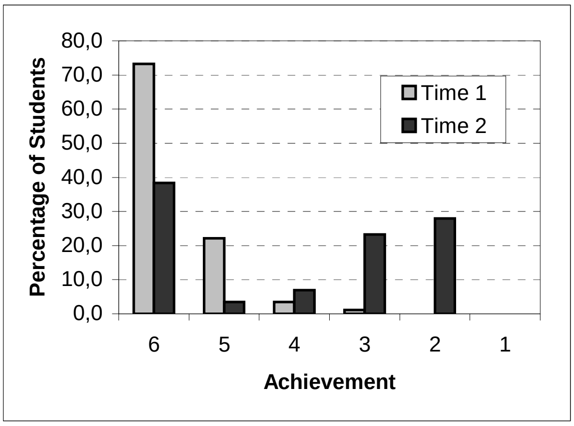

Figure 2. Frequency distribution of students' achievement at Time 1 and Time 2.

Working-memory capacity. The concept of working memory, that has been widely used in cognitive science, refers to the human limited capacity system, which provides both information storage and processing functions, and is necessary for complex cognitive tasks, such as learning, reasoning, language comprehension, and problem solving. The model was extensively developed by Baddeley and his coworkers [2]. The working-memory capacity of the students was assessed by means of the digit backward span (DBS) test, which is part of the Wechsler Adult Intelligence Scale. This test involves both storage and processing and has been used as measure of working-memory capacity in relevant works [6] [18].

Developmental level: Developmental level is a Piagetian concept and refers to the ability of the subject to use formal reasoning. It was assessed by the Lawson test, a pencil-paper test of formal reasoning [19].

3.2.2. *Hypotheses*

Four interdependent research hypotheses were associated with this work: 1) Problem solving is a nonlinear dynamical process. 2) The overload of the working-memory capacity could be understood better by reference to inhibitory processes. 3) Bimodal distribution of students' achievement could be modeled and explained by implementing catastrophe theory models. 4) Logical thinking could act as bifurcation parameter in a cusp catastrophe model, where the working-memory capacity acts as the asymmetry factor.

3.2.3. *Results*

The statistics of regression are summarized in Table 1. The cusp model proved to be superior to the linear pre-post control model explaining 69% of the variance. Large variance explained is not surprising for catastrophe theory models. Theoretically, in dynamical processes and in the vicinity of a bifurcation, a catastrophe theory model ignores the very large number of internal variables, and constrains the description of the local observed behaviour by a small number of control parameters [14]. This is the basic advantage for applying Catastrophe Theory for psychological and educational research.

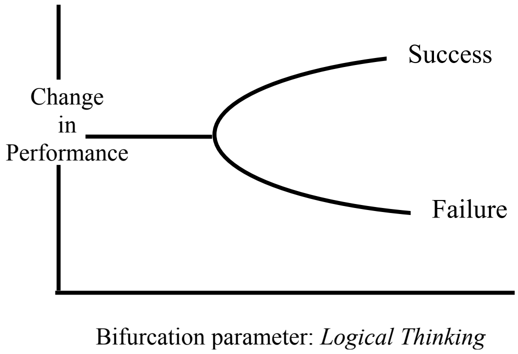

Figure 3. Bifurcation in students' achievements

Table 1. Summary of Regression for the Information processing: Cusp Catastrophe and Control Models (*N*=86). Working memory capacity (W) and Logical Thinking (L)

Model	Adj R^2	t	Model F
Linear 1	0.18		10.2****
W		2,88**	
L		-1,09	
Linear 2	0.17		6.7***
W		1,84	
L		-0,65	
M X L		0,13	
Linear 3	0.52		32.5****
Z_1		1,51	
W		5,57****	
L		-2,84**	
Cusp	0.69		47.6****
Z_1^3		3,41**	
Z_1^2		-4,45****	
$L X Z_1$		-2,67**	
W		5,42****	

*$p < 0.05$, ** $p < 0.01$, *** $p < 0.001$, **** $p<0.0001$,*

3.3. Model interpretation

The cusp model introduces *nonlinearity* to the behavioral data and to the theories existing behind the treatment. The model describes the pattern of behaviour (achievement) in problem solving, driven by two mental processes having functional relation with the chosen control parameters. It demonstrates that both linear and nonlinear changes in behavioural variable might be expected.

At low values of bifurcation variable changes are linear and smooth and at high values of bifurcation they are nonlinear and discontinuous. At low values of the asymmetry factor changes occur over the lower mode and are relatively small. At middle values of asymmetry factor, changes occur between modes and are relatively large. At high values of asymmetry factor, changes occur around the upper mode and are again small. At the control surface we can observe the bifurcation set mapping in the unfolding of the surface in two dimensions (Figure 1). The cusp bifurcation set induces two diverging response gradients, which are joined at a point, the *cusp point*. At the cusp point the behaviour is ambiguous, while the two diverging gradients represent the varying degree of probability that a student might succeed or fail.

The above geometry of behaviour suggests that for certain mental resources or variables involved in problem solving and in certain cognitive tasks of a given complexity, a point, the *bifurcation point* there exists, beyond which the system enters an area of discontinuous changes. Any subject with values of control parameters corresponding to points within this area named the area of *inaccessibility* could be pulled towards either attractor, success or failure (Figure 3). In addition, a phenomenon of *hysteresis* is observed, that is, subjects with the same parameter values oscillate between the two stable states, which also denotes the double *threshold* effect. The hysteresis introduces nonlinearity and demonstrates that small differences in control parameters, which are related to certain abilities or mental resources (initial conditions), may lead to sudden jumps from success to failure.

Generally, we can conclude on the initial research hypotheses that: 1) Problem solving is a nonlinear dynamical process. 2) The overload of the limited human channel capacity could be understood better by reference to disembedding ability or degree of field dependence/ independence, which corresponds to an inhibitory process. 3) Bimodal distribution of students' achievement could be explained by implementing a cusp catastrophe model. 4) Logical thinking acts as bifurcation parameter in a cusp catastrophe model, where the working-memory capacity is the asymmetry factor.

4. DISCUSSION: HIGHLIGHTS AND CONTROVERSIES

First, it is important to single out that a statistically significant cusp model introduces nonlinearity to the specific domain and to the existing theories. For cognitive science/ psychology the empirical evidence for hysteresis effect could provide bridges with Nonlinear Dynamical Systems Theory and support the dynamic nature of brain functioning and meets findings of important investigations taking place in contemporary neuroscience [20] [21]. For science education problem-solving education the nonlinearity has important theoretical and practical implications. First, in certain type of problems, which are non-algorithmic, novice problem solvers at a learning stage, follow mental paths and processes, which are nonlinear and dynamical in nature. In such problems the *information* required for the solution is not hidden exclusively in the initial conditions, but it is generated by the evolution of an iterative and recursive process [21] [22]. These are dynamical processes; the role of mental resources reflected in the variables implemented in this research is very crucial. This should have an impact on teaching where the algorithmic problem solving may be inadequate for preparing students to deal with unpredictability when they have to face unfamiliar and challenging cognitive tasks. Extended analysis of this matter could be found elsewhere [23]. Moreover, fostering the dynamical view, theories of learning, information processing and constructivism have to abandon the "computer metaphor" for brain functioning. Instead they have to develop more rigorous theoretical constructs and this is their future challenge.

There are some methodological issues and some controversies worth discussing about catastrophe theory and its application in social sciences. Since 1970, CT has used to describe discontinuous behavior at various circumstances such as Zeeman's proposed models [24]. On the other hand, it raised criticism [25] based on mathematical and epistemological arguments. Apart from the criticism, some Zeeman's models have properly been tested empirically [28]. Moreover, the development of mathematical background [34], statistical techniques and theoretical concepts, CT has been revisited and utilized in social science as a framework for modeling and understanding discontinuous behavioral changes in social systems.

In social science, there are three approaches in testing hypotheses based on catastrophe theory: The parameter estimation technique developed by Cobb [26] on the basis of the method of moments and maximum likelihood; the GEMCAT method of latent variable extraction by Oliva *et al.* [27] and the method developed by Guastello [28][29][30]. The latter is implemented in the present analysis.

There are some important remarks to be stressed about the later methodology, which is implemented in this research: The method is based on structural equations and uses polynomial regression techniques to test the catastrophe theory hypothesis. Basically it is a theory driven approach, where each control variables has an essential role within a theoretical model, which describes a process from where discontinuity emerges. The basic philosophy of this method is not the curve fitting, contrary to other approaches. Methodologically, in the regression procedure variables could be entered in the order of descending polynomials or simultaneously [31]. The experimental variables chosen for the model are not assumed to contribute to only one control parameter. On the other hand, it is important for the hypothesis tested, the relevance of the variable with the function of the variable to be specified.

The method implements dynamic difference equations, which operationalized differential functions, such as equation (4), staged as a probability density function (pdf) by using the Ito-Wright formulation [29] [31].

Another worth discussing point is the use of difference scores. The use of difference scores traditionally is not preferred in measuring psychological and educational data, because of the addition of errors. In classical psychometric theory, it is assumed that errors are uncorrelated with true scores. However this is not always true [32]. In intraindividual behavioural changes there are depended errors, which especially in the case of a nonlinear process, increase with the true scores. These 'errors' contribute to the expansion of variance. They are indicative of a nonlinear process and are captured only by nonlinear models [31]. Ergo, the catastrophe models explain a larger portion of the variance. But the main point, which is in the core assumptions of this methodology is that, whenever a difference score is utilized in psychological or educational data, a differential function is implied. This function accounts for the changes, which could be smooth and/or discontinuous changes.

There is a philosophical question related to the ontology of catastrophe models, and to whether these could be observed in social science data and in particular in the behavior of a cognitive system. This example of catastrophe theory application offers an interesting model for describing rather than explaining the behavior. This is a phenomenological model only; it cannot be said that it explains subjects' behavior. Zeeman has raised an essential question by asking what mechanism could be responsible for holding the state of the system on the behavioral surface [24]. The answer for a physical system, such as Zeeman's catastrophe machine, is nested in the operation of a potential function and the dynamic of the system.

In the mathematical formalism of the model (see corresponding section) an assumption was made, that is, the system is controlled by a potential function that is analogous to the potential function of an energy minimizing system. From a philosophical point of view, it is an important ontological question: How can one support the existence of such function and the existence of dynamics as well, in a psychological system (in favour to the application of catastrophe theory). We must point out here that the same philosophical question for the General Linear Model (GLM), that is compared as control in this methodology and it is so widely used in social science, is also unanswered. The assumption of linearity is the simplest thing to consider and to treat mathematically, but is it so in social and psychological reality? The addition, for example, of attitudes scores with IQ scores in a linear multiple regression, which examines the effect of some kind of attitudes scores and IQ scores on a behavioral variable, does not make much sense. What important information can the GLM tell us is, if a statistically significant (linear) effect of these factors on our dependent variable there exists. The linearity is one of the basic assumptions when the GLM is used.

On the other hand, nonlinearity consists a fundamental assumption in methodologies applied with analogous tools to brain-science [36], behavioral science [37] [38] [39] or to economics [40] and to catastrophe theory models as well [41] [42] [43]. The nonlinearity is the result of the operation of a potential function. It is important to support the 'existence' of such function and the existence of dynamic as well, in a psychological system. This is ontologically equivalent to the existence of the *attractor*, which is also fundamental in complexity theory and in dynamical system theory, and it is analogous to energy minimum is classical physics. The energy minimum is a special case of the *attractor* concept that is a stable state for system. In contemporary brain science and neuroscience, it has been shown that an observed pattern could be described as an "attractor", because the brain system is seen to converge to the pattern from a variety of starting points, as though it were "attracted" [44][45]. Thus, an obvious place to seek for attractors in a psychological system is the neural-base functioning of the brain. Moreover, a theoretical model [46] based on recent neuropsychological evidences has provided insights on brain functioning and a mathematical description of its dynamics. According to the model, brain functions as a dissipative dynamical system. This theoretical analysis provides support to the ontological assumption for the existence of 'potential' function and nonlinearity. According to the model, in compensation to unpredictability due to the nonlinear character of the underlying process, the following should hold for the system: The existence of multiple attractors possessing invariant measures in the dynamical system governed by the interplay among the order parameters and drastic reduction of degrees of freedom in the vicinity of a bifurcation and the emergence of essentially only a few dominant order parameters. The above support the dynamical hypothesis at the behavioral level and favor the application of catastrophe theory. Returning to behavioral level and behavioral science, given that brain functioning could give rise to attractors at this level, so that the operation of hypothetical potential function with multiple stable equilibria in a psychological system is not incompatible.

The theoretical analysis within a philosophical discussion answers or at least gives to some extent a satisfactory answer to the ontological question for nonlinearity. Nevertheless, further epistemological questions on methodology may arise at phenomenological level where the catastrophe theory models operate. It is promising, however, that CT and nonlinear methods in general, are receiving an increasing attention in social science research along with the development of the appropriate statistical and methodological tools.

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