NONLINEARITY, THRESHOLDS AND BIFURCATIONS IN KNOWLEDGE ACQUISITION AND PROBLEM SOLVING: TOWARDS A PARADIGM SHIFT IN EDUCATIONAL RESEARCH

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ABSTRACT

The present paper addresses the nonlinear dynamical hypothesis in knowledge acquisition and problem solving. These cognitive phenomena are processes of change or imply changes, which might be trivial, smooth or discontinuous. The usual linear approaches do not capture the dynamics of these processes. This work attempts first, to build theoretical bridges between the nonlinear dynamical systems (NDS) framework and the above cognitive processes, and second, to introduce nonlinear research methodologies. A Catastrophe Theory approach is proposed for testing nonlinear hypotheses in educational and pedagogical research. The data analysis involves dynamic difference equations and statistical regression techniques. Two examples from science education problem solving are provided, which implement cusp catastrophe models, accounting for discontinuities in students' performance. The meaning and the implications of the nonlinear model are discussed. Moreover, the perspectives for a potential paradigm shift in educational research methodologies are addressed.

1. INTRODUCTION

1.1. The Nonlinear Dynamical Hypothesis

The *Dynamical Hypothesis* (*DH*), which has been the central interest in cognitive and behavioural science for the last decades, appeared as an alternative to computational hypothesis [1]. The former originates from the development of connectionism, which brought mainly two changes in the traditional approaches: First, it altered the concept of *representation* of knowledge that refers to the internal structures, which encodes states of the world. The new concept, named *distributed representation* [2] suggests that the current internal state of representation can no longer be seen as a stable *repraesentandum* referring to a certain environmental state because it depends also on the previous internal states or activation patterns [3]. Second, connectionism permits modeling cognition as the behavior of dynamical systems. The essence of *DH* is encapsulated in the assumptions that cognitive agents are dynamical systems, and that the cognitive processes are considered as not discrete sequential manipulation of static representational structures (as the computer metaphor), but rather as processes unfolding in real time interacting and coevolving with the environment.

Changes in cognitive theories have direct implications to the field of education, since understanding of knowledge acquisition mechanisms leads to effective pedagogy. The traditional learning theories, such as behaviorism, information processing, constructivism and the biological approach, which have provided various metaphors as means to understanding learning, did not come to an integrated comprehensive approach. Nonlinear dynamical systems (*NDS*) theory offers a unified approach to describing and explaining phenomena involved in education and pedagogy [4], such as, learning, cognitive development and problem solving ability, which are actually nonlinear in nature.

The NDS theory provides concepts and principles, which could be utilized to explain changes in any system evolving in time. As system is taken to be any set of interdependent variables, which are entities having different values at different times and define the *state* of the system at a given time. Changes or transitions between states consists the *behavior* of the system. A system's behaviour could be understood via *self-organization* mechanisms, which comprise a conceptual framework for studying the spontaneous emergence of *order*, first applied to physical, chemical, and biological systems. The new paradigm involves fostering chaos and complexity as general theoretical frameworks [5] applied to physics, life sciences, psychology, and potentially to education and pedagogy. Some of the important concepts in NDS theory are the notions of nonlinearity, thresholds, bifurcations and attractors, which are also parts of the mathematical language. The next sections provide theoretical descriptions concerning the nature of knowledge, the process of learning and problem solving by means of NDS concepts.

1.2. Concepts as Attractors

In contemporary education, learning is named after the notion of conceptual change. Research in this area has shown that certain behaviours or students' 'alterative' ideas resist to change, thus making teaching and learning difficult. The theoretical frameworks, even the prevailing constructivism, have been mainly developed on metaphors and they have failed to provide an elaborated understanding about the nature of the phenomena involved in knowledge acquisition and the nature of knowledge itself. In the theoretical descriptions of cognition, artificial intelligence formalisms have introduced the classical division of hardware versus software for the distinction between architecture and processing. This partitioning is questionable, as far as the human brain is concerned, but it is still convenient for descriptive purposes.

The mind's architecture in the latest psychological theories concerns the nature of the representation or the format of knowledge. At a psychological level of description on this matter, knowledge is represented mentally in terms of *concepts*, *categories* and *propositions. Concepts* are the abstracted properties or attributes of objects or events; *categories* are the mental groups into which objects or events with common conceptual properties are placed; and *propositions* are entities that express relations among concepts. The above concern the declarative memory, which embraces both episodic and semantic memory and refer essentially to the knowledge contents of long-term memory. These theories attempt to provide the organization structures of the mind and the processes by means of which elements within this structure are brought into contact with each other. Structurally, semantic memory could be viewed or characterized as a hierarchical associative network of concepts [6].

Even a simple perception of a concept implies recognition of a category. Even though the lexical definition of a concept refers to recognition of a particular category, it appears that when relating to a concrete concept, one would be referring to a broader array of features or attributes many of which extended beyond the strict boundaries of the category. Ergo, *concepts* are not strictly circumscribed in their category definition. A way to include all features of a concept in an aggregate entity is to provide a kind of mapping. Extended forms of these maps are made when network architecture is added to the semantic mapping of concepts, which codes relation between concepts. In education, this representation of concepts, led to the development of the instructional and assessment instruments known as *concept mapping*. Concepts are nodes in a network and the links between nodes specify relations between concepts. These semantic representations permit neural networks to code and process simple proposition. Experimental research in neural networks has shown that propositional networks are able to simulate cognitive processes [7]. Learning is achieved when a specific weight attributed to links is modified by a given activation at the input nodes. The activation spreads through the whole network, modifies the weight of the links, and in addition renders the processing of subsequent inputs. In a neural network the neurons can activate or inhibit themselves. Thus, the network learns by means of a *self-organizing* mechanism.

On the other hand, in the human brain the creation process of such hypothesized hierarchical associative networks in memory and of course the manipulation of them in learning, thinking or communicating still remains an enigma.

Within the NDS theory many dynamical models have been proposed for the architecture of semantic networks, and how particular modules are selected and activated in the brain, while research at the behavioural level providing empirical evidence for the dynamic nature of semantic representations is very limited [8] [9]. A NDS approach has proposed associative networks of concepts as complex, organized semantic entities, *semantic constellations* (SeCos) of meaning that are dynamic in nature [10]. Hardy's semantic field model provides a rational description of how the semantic constellations bind together interconnected processes and semantic content. The cognitive architecture is based on diversified and specialized SeCos, where concepts are linked to a whole array of sub-cluster representing not only iconic or procedural schemes, propositional knowledge and language set, but also clusters of qualia, feelings, values, memories, desires and intentions. Thus, each SeCo is the locus of a unique self-organizing process where its elements are linked together and through experience cooperate and co-evolve. The *semantic linkages process* is a spontaneous nonlinear dynamical process. Depending on the manner in which they have been created, SeCos may demonstrate different levels of liability and adaptive capacity. A SeCo acts as an *attractor* that influences and organizes subsequent experiences. Note that the notion of attractor refers to stable equilibrium state of a system, thus is not easily susceptible to changes.

This point, from an educational perspective, is absolutely aligned with the notion of the ever-resisting students' 'alterative' ideas (or misconceptions), which inhibit conceptual change. However, this is more than an alterative metaphorical description. The NDS approach to knowledge-representation reveals a lot about the nature of knowledge-acquisition process. The semantic linkage process consists in changing an attractor. Attractors of a complex dynamical system are stable states of the system. Learning, as a modification process of an attractor is not a linear process of addition or substitution of element, but it is a nonlinear self-organization process, where changes in the entire attractor's layout are implied.

The thesis of the present paper maintains that for education research, issues on conceptual change should be revisited under the new NDS perspective. Learning as the process of changing or creating semantic networks, which behave as attractors of a complex dynamical system, should be viewed as a phase transition between stable states. A phase transition could be modeled as discontinuity, which implies the existence of thresholds and bifurcations. Thus, from research perspectives a different role of the variables should be assigned and different methodologies should be followed. A main issue to be resolved is the development of proper mathematical tools for detecting or inferring nonlinearity at the behavioural level.

1.3. Problem solving as a nonlinear process

Problem solving on the other hand, consists an important part in education, especially in science and mathematics. This section develops a theoretical analysis that aims to build bridges between concepts of NDS theory and problem solving, making so a contribution towards the development of a nonlinear theory in education. It is argued, that students or novice at a learning stage, when dealing with a certain type of problems, follow mental paths and processes, which are nonlinear and dynamical in nature.

It is important first to make a distinction between two broad categories, which all cognitive tasks can be classified into. In the first category are deductive-type cognitive tasks whose solutions always exist. The outcomes of these problems are unique and are implicitly contained in the initially given data set. During such processes no new *information* is produced [11]. In the second category are the inductive in nature tasks whose solution may not be unique or may not exist at all. The outcome in this type of problem solving is not guaranteed from the subject's previous experience or his/her existing repertoire. For the inductive-type complex problems where multiple solutions might exist, there is no unique path to follow and each step is determined by the previous steps. The *information* required for the solution is not hidden exclusively in the initial data, but is generated by the evolution of an iterative and recursive process. These are dynamical processes, whose outcome can be understood in an evolutionary context.

In education, an analogous distinction is made between cognitive tasks called *exercises* and (*"real"*) *problems*. Exercises are mental tasks where the subject applies a well-known procedure, which has usually been practised. Reaching the final outcome-solution is credited to the student's ability to execute successfully the learned algorithm. On the other hand, 'real' problems or complex problems which are not algorithmic, *mimic* or *simulate* inductive-type problems, when faced by a novice, in the sense that for him/her the outcome is not obviously nested in the initial conditions/given data, and the solution is not guaranteed by the students' previous repertoire. The strategy to be followed is not known and the outcome is in part generated by the evolution of the mental process. We must emphasize that in a school context a cognitive task can be an exercise or real problem depending on the subject's expertise and on what had been taught. A task could be an exercise for a student having a certain repertoire, while the same task is a problem for another student [12].

From NDS view the cognitive process is driven by the action of mental resources corresponding to certain cognitive variables, which could be measured in experimental psychology and have been proved to play an important role in certain mental tasks. Interdisciplinary research has shown that cognitive variables, such as, working memory capacity, disembedding ability, logical thinking, mental capacity and mobility-fixity dimension have proved to be predictive for students' performance. These may belong to different theoretical constructs either to information processing models or to neo-Piagetian theories.

Let us see how the action of certain mental constructs from neo-Piagetian theories could be viewed within the NDS framework. In a cognitive task execution, the mind activates certain operative schemes, which are responsible for the transformation and coordination of the preexisting or the new information (or schemes/knowledge) with the action of variety of mental resources named *constructive operators* each of which performs a specific class of function [13]. For example, there is the *M*-operator for information processing capacity, the *F*-operator for disembedding ability, the *L*-operator for logical thinking and so on. One could recognize a number of mental subprocesses corresponding to the action of the above operators. Thus, in a mental- task execution, a mind might proceed as follows: inputs data, retrieves information from long term-memory, processes information, separates "signal" from "noise" (if any), applies formal reasoning, processes information again and so on.

In executing a familiar algorithm, the sequence of the above subprocesses is predetermined. The successive activations of various schemes and operators follow a step-by-step linear or cyclical procedure. The solution is nested in the algorithm and in the initial conditions/data. It is imperative to say that in these processes a student's (or novice's) mind may even be ignorant of the strategy followed or of how to turn the implicit into explicit.

On the other hand, in a 'real' problem situation, the sequence of the above steps is not predetermined. The mind might activate successively the above subprocesses, in a pattern that is neither linear nor cyclical. This pattern emerges from an interwoven interaction among mental operators; it might be seemingly random and might exhibit self-similarity and fractal structure. The successive mental functions are selected by a random choice (microscopic fluctuations) with no preexisting scenario. In such dynamical process both convergence and divergence are present. The interplay between convergence and divergence, analogously to the role of competence versus cooperation in perceptual motor learning [14], determines the emergence of the final outcome. An underlined *self-organization* mechanism is implied in such processes as the *synergetic* view to intelligence proposes [15]. For a cognitive system this *self-organization* is essentially a *learning* process. Until performance converges to a solution, that is a point attractor (in the language of dynamics), it may go through a more complex dynamics, even chaotic, resulting from the nonlinear interactions between the cognitive task and the subjects' mental resources. Given that, each step is determined by the previous steps, there is no *a priori* truth to be revealed and the process becomes history dependent (in Prigogine's sense). Thus, one might expect that at certain points in time, the trajectory of the system in state space bifurcates into multiple asymptotically stable (attractors) or unstable (repellers) regimes in ways depending on critical values of the control parameters [16]. One might expect ineffective repeated attempts on the way that correspond to limit-cycle attractors, and may prevent the evolving trajectory (scenario) from moving forwards.

The dynamic problem solving process could be seen also as following a punctuated equilibrium model analogous to the evolutionary dynamics seen in genetic systems, where long periods of stasis are alternated by short periods of rapid changes. These changes are phase transitions and could be modelled as discontinuities or catastrophes. Variables associated with mental resources involved in problem solving can control the transition. If a catastrophe occurs then the trajectory may fall into a less optimum attractor, which did not facilitate further proceeding towards the solution. The trajectory possesses a plethora of successive bifurcation points until the process converges. A cascade of bifurcations has a cumulative effect on the system's state and it might cause the mind to proceed successfully, to halt or be misled and trapped into an attractor (of low achievement). This cumulative effect is what one may evaluate on an examination paper with a final mark.

In conclusion, learning and problem solving for novices are nonlinear dynamical possesses. NDS theory can provide a better framework for understanding these cognitive processes. Learning, in many previous research works [14] [17] [18], has been hypothesized as nonlinear dynamical process, even chaotic, evolving via self-organization mechanisms. It may involve analogous mechanisms with those in learning-task procedures having dynamic characteristics reflected in the common learning curves [18] [19].

Fostering the NDS framework in behavioural science has an immediate impact to educational theories. From a theoretical point of view, it is unambiguous that linear methodologies are inadequate for describing any complex system's behaviour, because of the nonlinear interactions among variables. The nonlinear interactions might lead to the existence of discontinuities, thresholds and bifurcations where the role of each variable might have changed. The theoretical framework in education and pedagogy requires also the development and the implementation of nonlinear mathematical methods for testing *Nonlinear Hypotheses* in research. A promising perspective on this development is Catastrophe Theory (CT), which provides the proper conceptual framework for modeling discontinuities in a system's behaviour as well as the mathematical tools to describe these changes.

In the following sections elements of CT and two applications in educational research are provided.

2. CATASTROPHE THEORY

2.1. Mathematical aspects

Catastrophe theory [20] concerns the study of equilibrium behaviour of a larger class of mathematical system functions that exhibits discontinuous changes. It relates discontinuous changes in dependent variables as a function of continuous variation of the independent variables (controls). CT models in science involve dissipating systems or potential-minimizing systems. Such models ignore the very large number of internal variables, and they constrain the description of the local observed behaviour by a small number of control parameters [21] [22] [23].

An important aspect of CT is the classification Theorem, which states that all discontinuous changes of events can be modeled by one of seven elementary topological forms [24]. These forms are hierarchical and are described by one to four control parameters depending on the complexity of the behaviour they encompass and describe. The elementary catastrophe models are classified into two groups: The cuspoids and the umbilics. The formers have drawn most of the attention in social science applications. They involve one dependent variable and have potential functions in three to six dimensions and response surfaces in two to five dimensions. The potential function is the integral of the response surfaces function. They are namely the fold, cusp, swallowtail and the butterfly model [24]. The potential function for the fold catastrophe is

$$V(y,a) = y^3/3 - ay \quad (1)$$

Where y is the dependents measure and a is the control parameter. Its response surfaces function is defined as the set of points where the equation (2) holds:

$$\delta V(y,a)/\delta y = y^2 - a \quad (2)$$

The most interesting and most applicable is the cusp catastrophe model. The cusp model applies to a system that has two states of stable equilibrium or two *attractors*. It describes changes between two qualitatively distinct forms for behaviour or states. These states within the context encompassed in an educational system could represent *success* or *failure*.

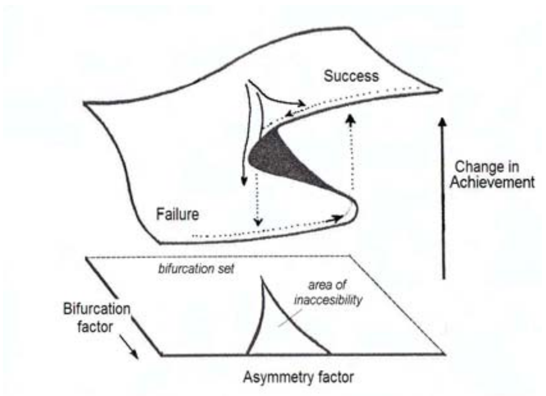

Figure 1. A three-dimensional representation of the cup catastrophe response surface for students' achievement.

Changes between the two states is a function of the two controls: asymmetry (a) and bifurcation (b). a and b are the independent variables and have a particular function in the process of change and in the underlying mechanism. The relation between behaviour y and the controls is given by the equation (3), which corresponds to potential function of the cusp catastrophe:

$$V(y, a, b) = y^4 / 4 - by^2 / 2 - ay \tag{3}$$

While the cusp response surface is a set of points where

$$\delta V(y, a, b) / \delta y = y^3 - by - a \tag{4}$$

The cusp surface is three-dimensional and features a two-dimensional manifold (Figure 1). For a dynamical system controlled by a potential function the equation dy / dt = - δV / δy holds. Thus, the set of values of y defining the response surface can describe changes in behaviour over time. A description of the cusp response surface is given in section 3.4.

The swallowtail model describes a behaviour consisting of two stable states and two unstable areas and it is described by three control parameters. The butterfly catastrophe involves four control parameters and it describes a spectrum of behaviour consisting of three qualitatively different stable states. More about CT could be found elsewhere [21] [22] [24].

Some important concepts associated with the implementation of CT, especially with the cusp model are the notion of *random force*, which occurs when the system is exposed to an entropy-inducing event (in thermodynamic sense). As the entropy of the system increases, the probable location of its elements in space expands to the limits of its confinement. Bifurcation can serve as entropy reducing mechanism by partitioning the elements into two areas of relative stability. The elements of the system are not equally exposed to random force because another parameter determines the level of expose and this is the bifurcation parameter in a cusp model [24]. In these nonlinear processes the concept of *causality* differs from the traditional view. In CT model the coaction of random force and bifurcation mechanism are considered jointly and are said to cause the phenomenon, which implies bifurcation, entropy and autonomous process. The concept of cause is replaced by a combination of *control*, and the usual independent variables are referred as controls.

2.2. CT: From Science to Science Education

Catastrophe Theory has been applied to many fields. In physics and engineering models have been developed for the propagation of stock waves, the minimum area of surfaces, nonlinear oscillations, use of catastrophe topology for conceptual formulation of thermodynamics, scattering and elasticity. An interesting and instructive example in science is the predictions of van der Waals equation in the transition between the liquid phase and the gaseous phase of matter. In this approach, the van der Waals equation is rewritten as cusp catastrophe with the temperature and pressure as conflicting control parameters. The temperature is the asymmetry factor and pressure is the bifurcation factor, while the density is the dependent variable. On the cusp response surface (analogous to Figure 1), the top sheet (upper mode) is the liquid phase and the bottom sheet (lower mode) is the gaseous phase; the two catastrophes represent boiling and condensation. The vertex of the cusp is the critical point, where liquid and gas coexist. Going around the back of the cusp represents sublimation, where the liquid can be converted into gas without boiling [21].

Applications of catastrophe theory have been reported in biological science [25] and also in behavioural and social science, from psychology [19] [24] to economics [26]. Other work, such as research on parameterization of learning curves [27], training and program valuation [28] and motivation and academic performance [28], is worth mentioning.

Catastrophe theory has been connected to Piagetian stagewise development and some interesting models have contributed to the development of this area. A theoretical cusp model [30] showed that balancing *assimilation* and *accommodation* could lead to sudden jumps to a new stage in development. A related experimental work [31] showed that a catastrophe model involved discontinuities in the responses of children and described the transition from preoperational to concrete operational thought. An overview of the traditional methodological approaches concerning stagewise cognitive development and their integration on the basis of catastrophe theory suggested cusp catastrophe models for stage transition by employing control variables from neo-Piagetian theories [32].

Science education research, within the neo-Piagetian framework, has provided an empirical catastrophe theory model for problem solving which accounted for the discontinuities observed in students' performance, [33] and sets the framework for the application of catastrophe theory in educational research.

3. EDUCATION RESEARCH APPLICATIONS

3.1. Methodological aspects for a cusp model

In this section the application of catastrophe theory is described. The method presented here involves dynamic difference equations and statistical regression techniques. The dynamic difference equations apply where the behaviour of a system is measured at two points in time. During the time that elapses between the two measurements, time 1 and time 2, it is assumed that a 'random force' has been applied to the system. Consider at time 1 a system where the dispersion of its elements (depended variable) follows a unimodal distribution [24] [28] [29]. The application of the 'random force' is assumed to be sufficient to excite trajectories across the fullest possible range of the catastrophe manifold and to result a bimodal distribution (Figure 2, a schematic diagram). The bimodality is not necessarily due to the presence of two distinct subpopulations, but it may reflect a nonlinear system having double stable equilibria, that is, two *attractors*. For an educational research during the elapsed time between measurements an intervention or the action of mental resources is assumed. These mental resources are correlated with the variables, which could be implemented as controls in a cusp model. The method is developed for testing theory driven hypotheses and can support bridges between NDS and any behavioral science theory.

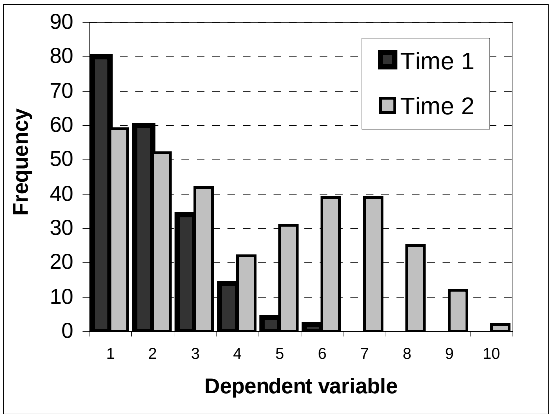

Figure 2. Bimodality in the dependent variable of a system.

For the statistical analysis, the raw scores of the dependent and independent measures were transformed to Z scores corrected for location and scale σ_s.

$$Z = (Y - Y_{\min})/\sigma_s \quad (5)$$

The location correction is made by setting the zero at the minimum value of Y. The scale, σ_s, represents the variability around the modes, which is different from the ordinary standard deviation (σ). σ is sometimes inadequate measure of the scale. A nonlinear model is very sensitive to location and scale, in contrast to the linear models.

The specific equation to be tested for a cusp catastrophe model is:

$$\Delta Z = Z_2 - Z_1 = b_1 Z_1^3 + b_2 Z_1^2 + b_3 F Z_1 + b_4 M + b_5 \quad (6)$$

The alternative linear models are the following:

Linear 1, $$\Delta Z = b_1 M + b_2 F + b_3 \quad (7)$$

Linear 2, $$\Delta Z = b_1 M + b_2 F + b_3 MF + b_4 \quad (8)$$

Linear 3, $$Z_2 = b_1 M + b_2 F + b_3 Z_1 + b_4 \quad (9)$$

The cusp model holds if equation (6) proved to be superior to the linear alternatives in terms of statistical indices and variance explained.

3.2. A cusp model for information processing

Science education researchers in order to understand and explain students' performance have fostered psychological theories that focus on the capacity limitation of short-term memory, which has been proved to be of great importance mainly in problem solving. A particular model stated that if a student does not fail for lack of information or recall, he/she is likely to be successful in solving a problem if the problem has a mental demand, which is less or equal to the subject's information processing capacity, unless the student has strategies that enable him/her to reduce the mental demand of the problem to become less than his/her processing capacity. Mental demand of a problem is associated with the number of steps required for the solution. As a problem increases in complexity and if the human channel capacity has a final limit, sudden decrease of achievement is expected. The approaches implemented in these information-processing models were based on the assumption of linearity, and included only one variable, while it has been shown that the interference of other variables, such as the degree of field dependence/ independence and the critical thinking play an essential role [35] [36]. However, the linear methods failed to provide an explicit model for the sudden decrease in students' performance, while they did not use the proper mathematical tools to account for discontinuities. An interesting attempt was the development of nonlinear models [37] [38] [39], which have provided a description of changes in students' achievements within complexity theory as phase transitions.

Within catastrophe theory conceptual framework and implementing the proper mathematical tools a nonlinear information-processing model was developed, which accounts for the discontinuous changes in student's performance in problem solving. The proposed cusp catastrophe model utilizes two controls: Functional M-capacity (M) as asymmetry and field dependence/ independence (F) as bifurcation parameters [34]. Functional M-capacity (M) belongs to Pascual-Leone, theory of constructive operators and is a measure of the information processing capacity of the subject. M-capacity was assessed by means of the Figural Intersection Test (FIT) [40]. Disembedding ability or field dependence/ independence is the ability of the subject to separate 'signal' from 'noise', and it is assessed by means of the Group Embedded Figures Test, GEFT, [41]. Science education research has verified that field dependent students appear to possess lower information processing capacity and demonstrate lower achievement comparing to their field independent peers. Thus, disembedding ability has been associated with the information processing, but in the linear models non-explicit role has been assigned to this variable.

Measures at time 1 (Test 1) included knowledge recall questions from the related theory, simple calculations or partial steps and simple conceptual questions. The above assured that a minimum of basic prerequisite knowledge was available. Measure at time 2 (Test 2) was a high-demand problem of 7-step solution. Test 2 presupposes Test 1 and requires the action of mental resources related to cognitive variables (M-capacity and field dependence/ independence) implemented as controls. Time is implicit here.

The cusp model proved to be superior to the linear pre-post control model explaining 77% of the variance. Table 1 shows a summary of regression for the cusp catastrophe and the control models. Large variance explained is not surprising for catastrophe theory models. In dynamical processes and in the vicinity of a bifurcation, a catastrophe theory model ignores the very large number of internal variables, and constrains the description of the local observed behaviour by a small number of control parameters [21]. This is a very promising aspect of Catastrophe Theory for psychological and educational data.

Table 1. Summary of Regression for the Information processing: Cusp Catastrophe and Control Models (*N*=124).

Model	Adj R^2	*t* (weight)	Model F
Cusp model	0.77		106***
Z_1^3		3.75**	
Z_1^2		-5.28***	
F X Z_1		-5.55***	
M		4.74***	
Difference Control	0.04		2.63*
M		-1.43	
F		-1.88	
M X F		0.80	
Pre-post Control	0.46		36.9***
Z_1		1.2	
M		5.30***	
F		-6.52***	

*p < 0.05, ** p < 0.001, *** p<0.0001.

3.3. A cusp model for higher-order cognitive skills

An interesting area in science education problem solving deals with teaching higher-order cognitive skills (HOCS). HOCS referred to high demand problems, which require among others analysis, synthesis, decision-making, critical thinking and problem-solving capabilities [42]. These cognitive tasks, while presupposing basic knowledge in a specific domain, require computational/algorithmic procedures and in addition a deeper conceptual understanding. The basic knowledge assessed as recall question from related theory, simple calculations and simple conceptual questions, is referred as low order cognitive skills (LOCS). There is a extensive discussion in this area about conceptual understanding versus algorithmic problem solving in chemistry education. The traditional methods of teaching relay mostly on computational exercises, where the learners are expected to acquire expertise in the domain by practicing well-known algorithms. It has been pointed out that this learning style is not compatible with attaining conceptual learning and utilizing HOCS. On the other hand, the conceptual understanding has been proposed as distinct from algorithmic problem solving ability, thus, competence in the former is not connected with the competence in the latter.

The present application proposes a cusp catastrophe model where the change in students' performance is measured as the difference between the normalized achievement scores in HOCS minus LOCS, and it is described as a function of two controls: The algorithmic problem solving ability (A) as asymmetry and the conceptual understanding (C) as bifurcation factor. The variables, A and C, were assessed separately by analogous questions and problems. The HOCS test presupposes LOCS test and requires computational abilities demonstrated in algorithmic problem solving, and conceptual understanding.

Table 2 shows a summary of regression for the cusp catastrophe and the control models. The results support the cusp catastrophe model explaining 89% of the variance. The model suggests that dealing with HOCS is a nonlinear process involving mental resources related to computational abilities on one hand and to conceptual understanding as well. Conceptual understanding acts as bifurcation variable, that is, a threshold there exists beyond which small variations in the related mental resources can cause a nonlinear shift from success to failure.

In addition, the above abilities/variables, in the cusp model operate as opponent processes, and thus are proved to be different in nature as it has been proposed elsewhere [43] [44] and they could interact in a nonlinear fashion resulting discontinuous changes in students performance.

Table 2. Summary of Regression for HOCS: Catastrophe and Control Models (*N*=251).

Model	Adj R^2	t (weight)	Model F
Cusp model	0.89		131***
Z_1^3		5.67***	
Z_1^2		-8.52***	
C X Z_1		-5.37***	
A		9.10***	
Difference control	0.24		59.8***
A		1.75	
C		-0.42	
A X C		2.36*	
Pre-post control	0.58		91.2***
Z_1		-19.9***	
A		5.85***	
C		-4.31**	

*p < 0.05, *** p<0.0001.

3.4. The meaning of the cusp catastrophe model

The cusp model introduces *nonlinearity* to the behavioral data and to the theories existing behind the treatment. The model describes the pattern of behaviour (achievement) in problem solving, driven by two mental processes having functional relation with the chosen control parameters. It demonstrates that both linear and nonlinear changes in behavioural variable might be expected.

At low values of bifurcation variable changes are linear and smooth and at high values of bifurcation they are nonlinear and discontinuous. At low values of the asymmetry factor changes occur over the lower mode and are relatively small. At middle values of asymmetry factor, changes occur between modes and are relatively large. At high values of asymmetry factor, changes occur around the upper mode and are again small. At the control surface we can observe the bifurcation set mapping in the unfolding of the surface in two dimensions (Figure 1). The cusp bifurcation set induces two diverging response gradients, which are joined at a point, the *cusp point*. At the cusp point the behaviour is ambiguous, while the two diverging gradients represent the varying degree of probability that a student might succeed or fail.

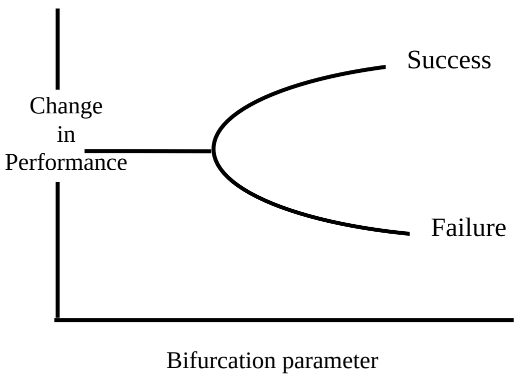

Figure 3 Bifurcation in students' achievements

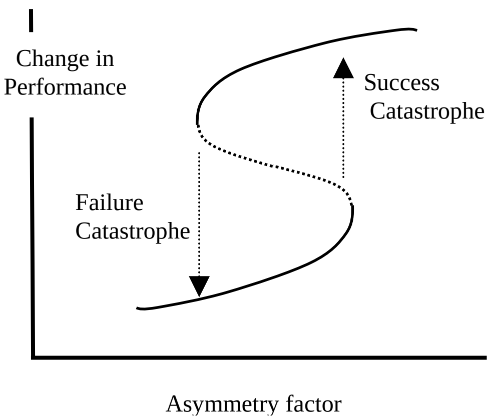

Figure. 4 An example of hysteresis effect for students' achievement.

The above geometry of behaviour suggests that for certain mental resources or variables involved in problem solving and in certain cognitive tasks of a given complexity, a point, the *bifurcation point* there exists, beyond which the system enters an area of discontinuous changes. Any subject with values of control parameters corresponding to points within this area named the area of *inaccessibility* could be pulled towards either attractor, success or failure (Figure 2). In addition a phenomenon of *hysteresis* is observed, that is, subjects with the same parameter values oscillate between the two stable states and also denotes the double *threshold* effect. The hysteresis introduces nonlinearity and demonstrates that small differences in control parameters, which are related to certain abilities or mental resources (initial conditions), may lead to sudden jumps from success to failure (Figure 4).

4. NEED FOR THE PARADIGM SHIFT

In educational research methodologies, the basic assumption in both quantitative and qualitative approaches is the linear cause-and-effect relation between dependent and independent variables. Modeling based on linear statistics is not limited because the lack of normal distributions, which is ameliorated by non-parametric tests, but mainly because the linear paradigm offers a simplification, which is inadequate for developing any theoretical framework in psychological and educational science. The system under investigation in this area, a child, a class or a school unit or even the educational system as a whole is a complex system evolving in time. Thus, the variability in any dependent or independent observable factor cannot be treated as 'error' around the mean. The multimodal probability density functions often met are evidence for the existence of fundamentally nonlinear underlying stochastic processes and reflect a nonlinear dynamical system having multiple stable equilibria. The conventional approach reduces the systems' behaviour to the function of its variables, ignores any nonlinear interaction among them and fails to explain discontinuous changes.

Research in education and pedagogy aims to provide a better understanding about the factors affecting students' performance and to suggest the optimum means for the preparation of students for productive functioning in a highly demanding and continuously changing environment. In research and practice, what are actually measured, are changes: Changes in achievement, changes in any behavioural variables involved in educational setting, which might concern individual or group performance. Education is a process of change. The change might be trivial, smooth or discontinuous. A discontinuous change may imply a qualitative change. The usual employed linear models have poor explanatory power. The linear, common sense cause-and-effect paradigm does not always hold when applied to the above settings. Knowledge or expertise acquisition and cognitive development are more complex processes when considering the variety of ability levels, patterns of thinking, learning styles, personality traits, and cultural backgrounds. The composite student profile is not a simple weighted linear sum of the contributed components. These components are rather interacting in task executions, with each other in a nonlinear fashion, and this could lead to surprises and unexpected, successful or failed outcomes. Additional complexity is introduced by the function the 'non-intellective' characteristics, such as motivation and self-regulation.

It must be emphasized that in education, besides constructive variables, such as natural gifts/mental resources or effective learning settings, there are inhibitory variables and processes originating from negative attitudes or hostile environments. A linear treatment of such variables hardly provides a satisfactory description of reality, because it cannot capture the dynamics of the change processes. The latter could be overcome by fostering Catastrophe theory approach, which would have multiple implications for any cognitive and developmental psychological theory that have an impact on pedagogy. Novel theories, have already promoted the idea of both smooth and/or discontinuous changes, but no empirical evidences have been provided.

According to the thesis of this paper, a new area of investigations could open in educational and pedagogical research considering the plethora of the unresolved hypotheses, which could be revisited under the new perspective of NDS theory and the feasibility of testing nonlinear hypotheses provided by CT. The educational research is not restricted to cognitive or developmental theories. Further investigations could be realized in social approaches, cooperative learning, applications with information and communication technologies, and in educational management as well.

In conclusion, changes in education and pedagogy cannot be effectively studied under the conventional reductionistic view of mechanistic models. Ignoring the dynamic nature of the processes involved, assumptions of linearity in both quantitative and qualitative approaches, may falsify our estimations in the degree of change (assessment). But the most important point is, that it may falsify our estimations in the pattern of change as well, which is a prerequisite for improving educational practices. Ergo, education and pedagogy should be close to making the shift of paradigm, which has already been fostered in other behavioural sciences.

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