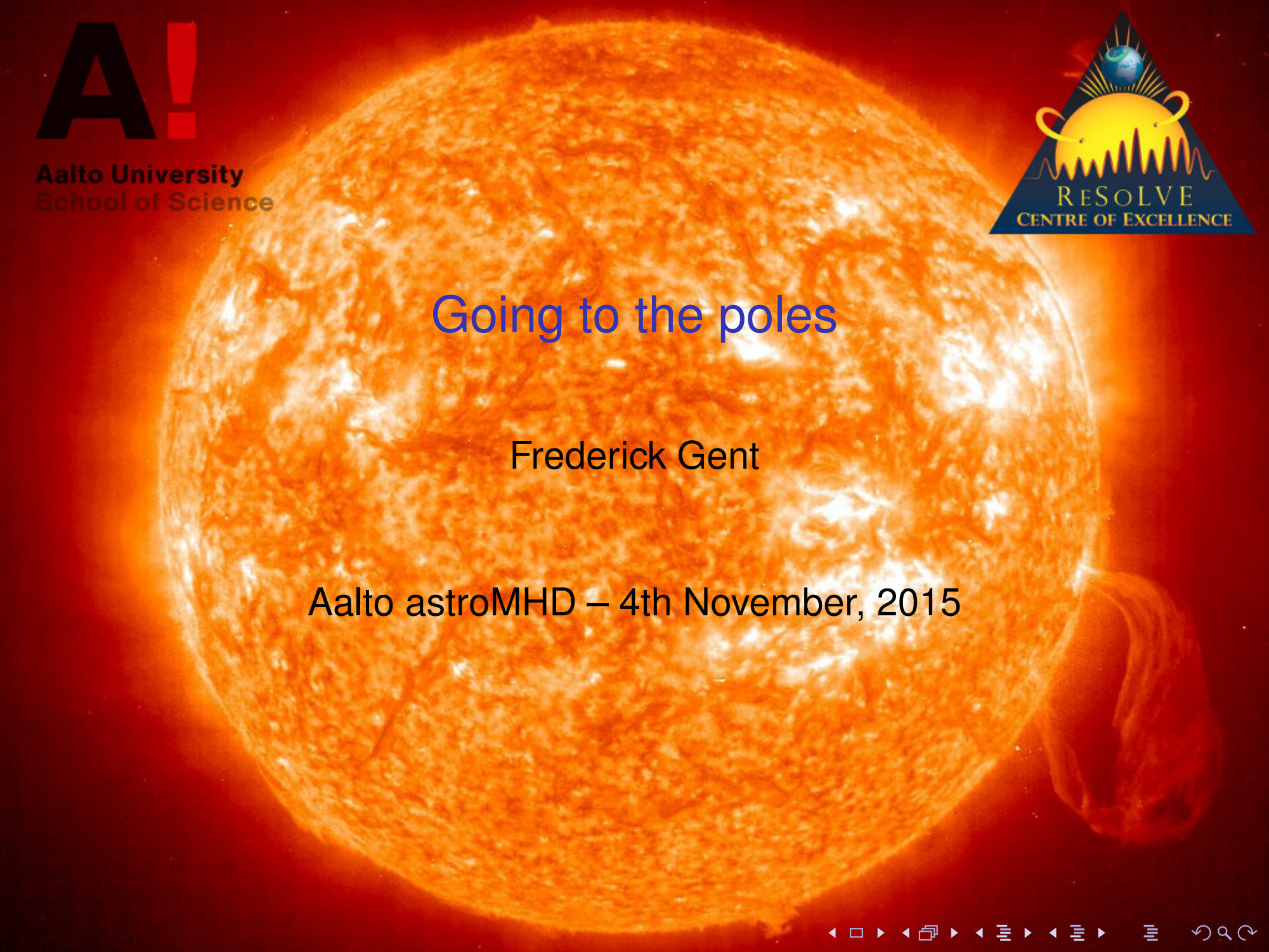
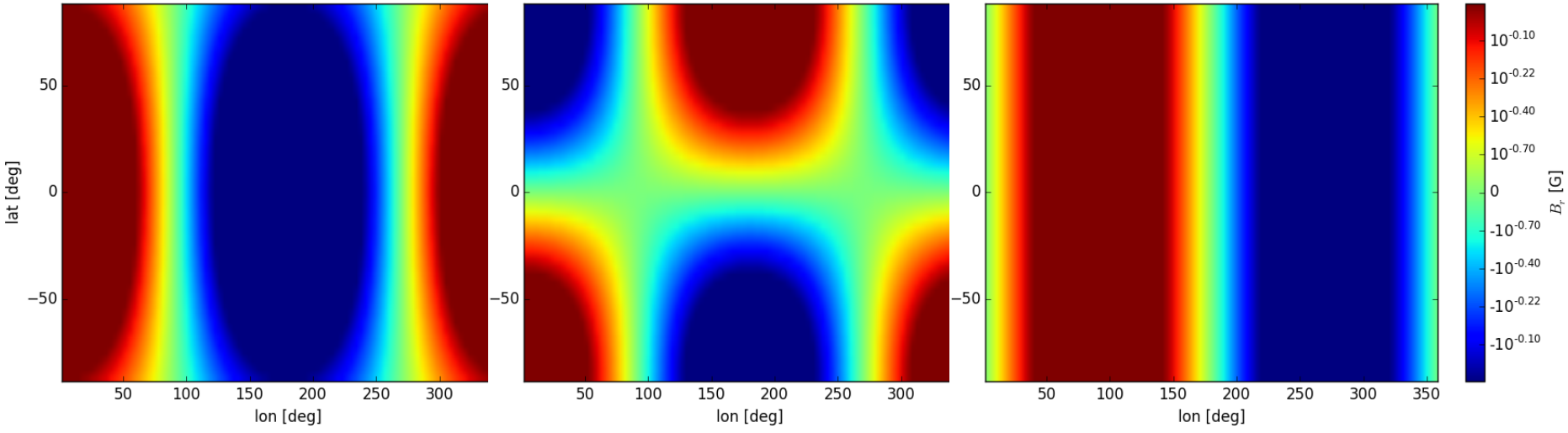


Going to the poles

Frederick Gent

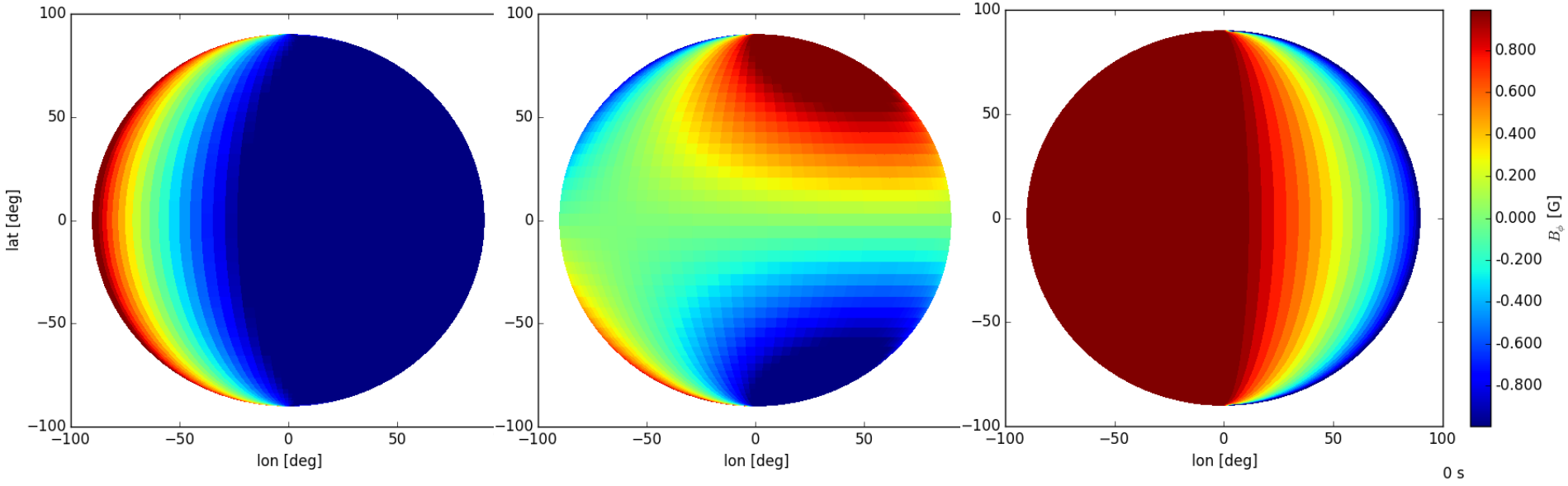
Aalto astroMHD – 4th November, 2015

Horizontal vs vertical dipole



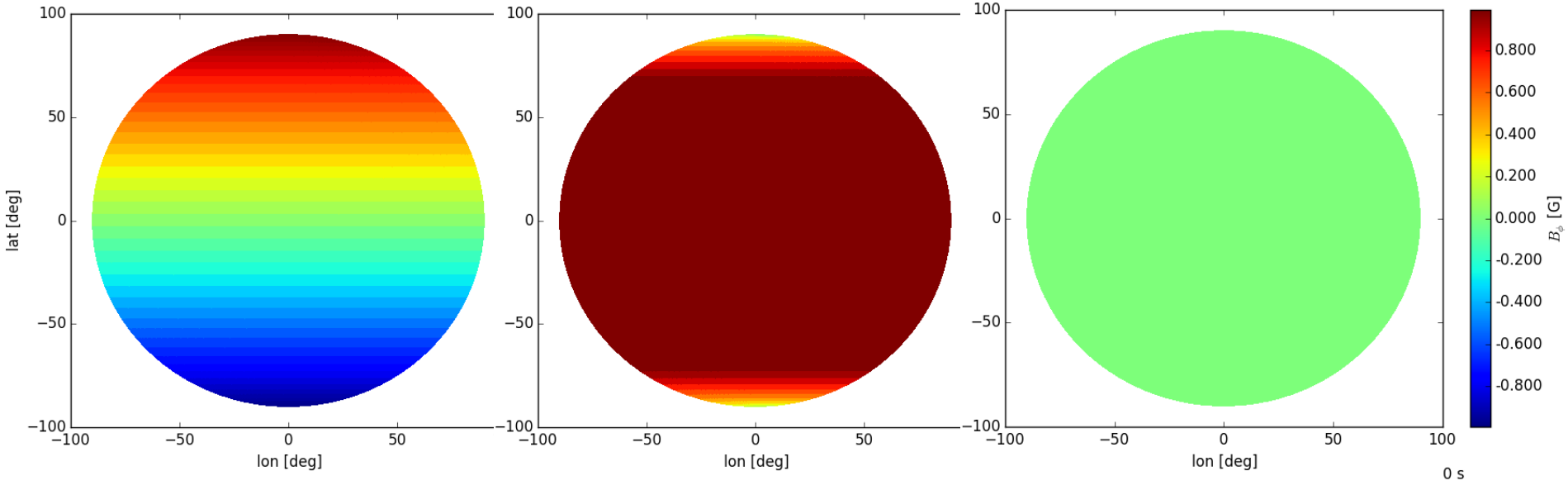
$$\boldsymbol{A} = -\boldsymbol{r} \times \nabla \Psi, \qquad \Psi = f(r) \sin\theta \cos\phi, \qquad f = r^2.$$

Horizontal dipole



Radial boundary conditions on ***A***: stress free 'sfr'

Vertical dipole



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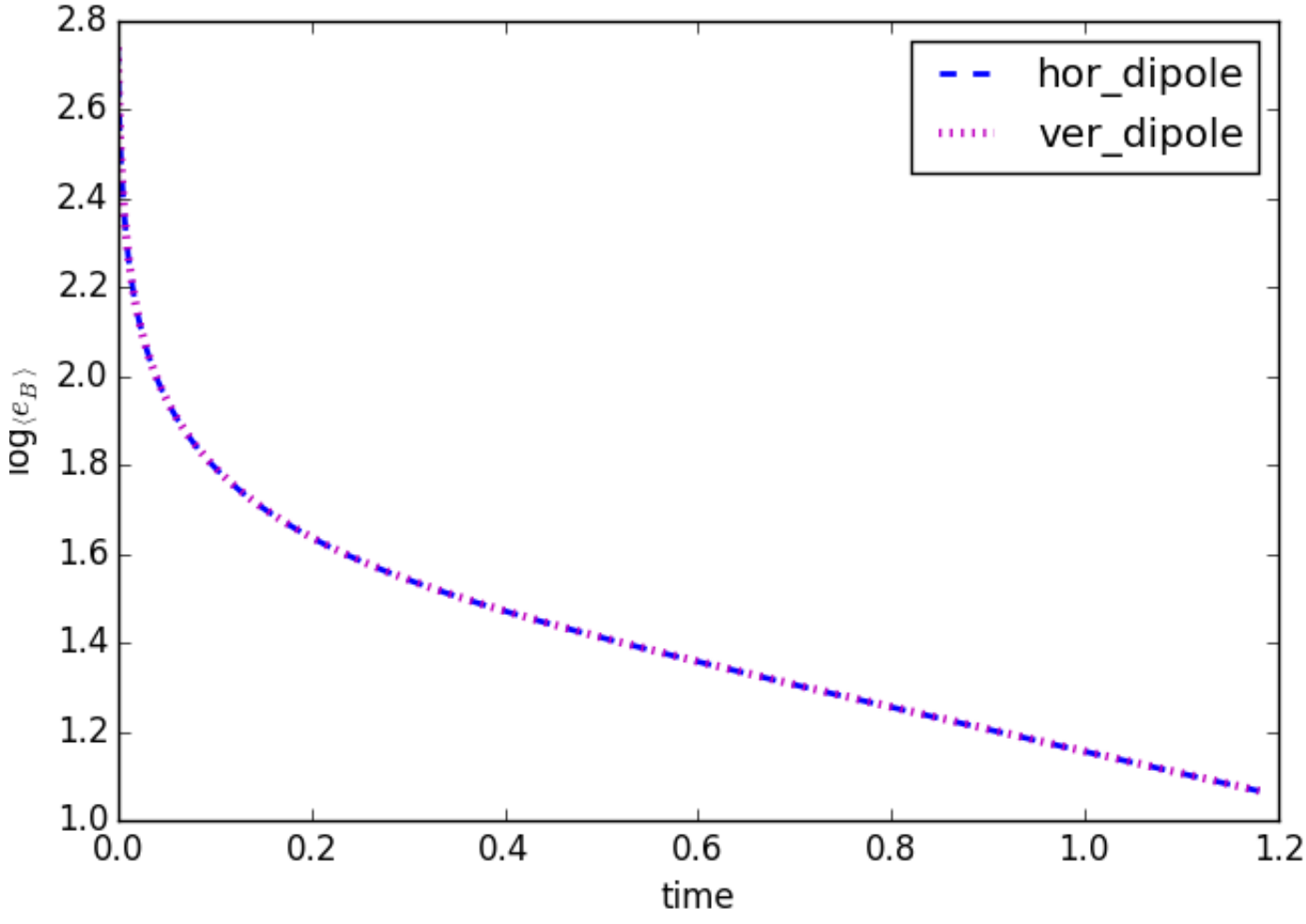
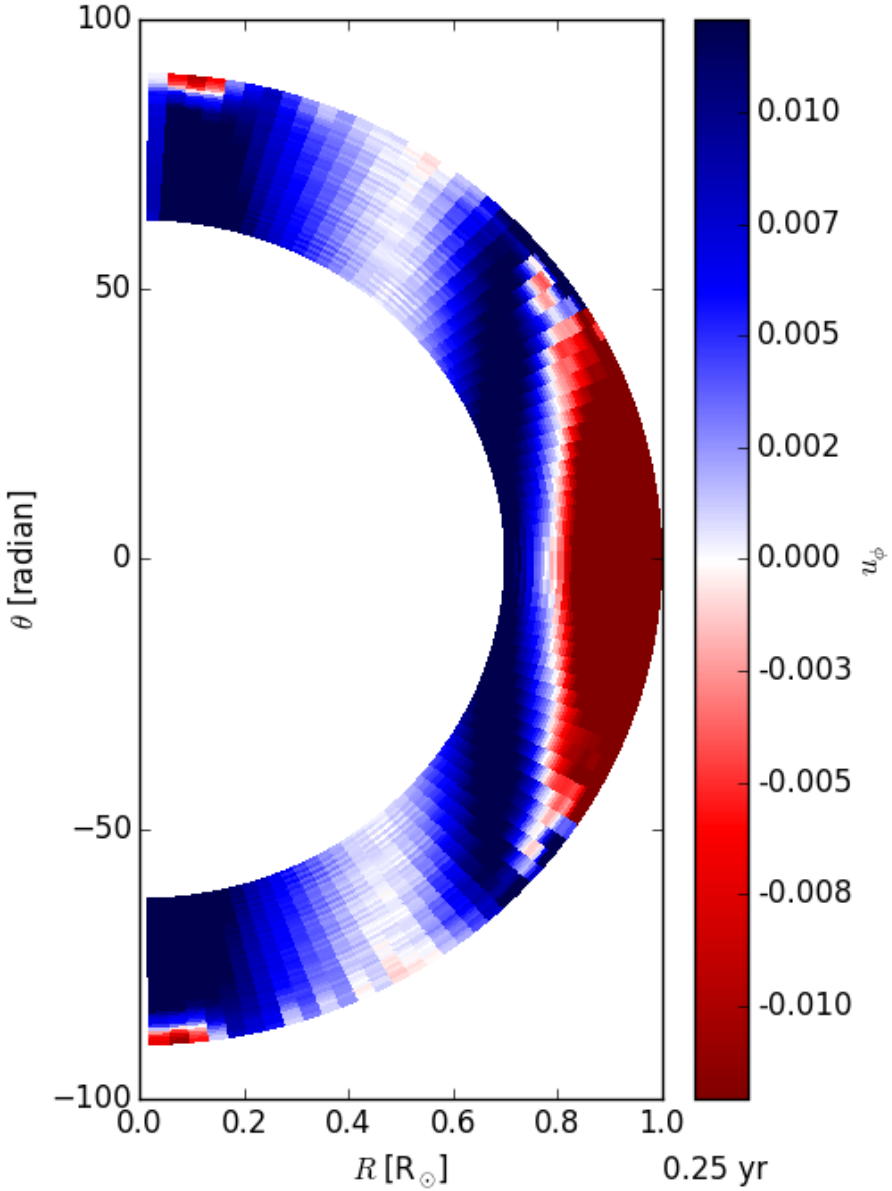
Decay rates for horizontal or vertical dipoles



$$\frac{\langle e_B \rangle_{\text{hor}} - \langle e_B \rangle_{\text{ver}}}{\langle e_B \rangle_{\text{hor}}} =\simeq 0.0006,\ dx = 0.006,\ d\theta = d\phi = 0.065,\ dt = 4.255 \cdot 10^{-5}$$

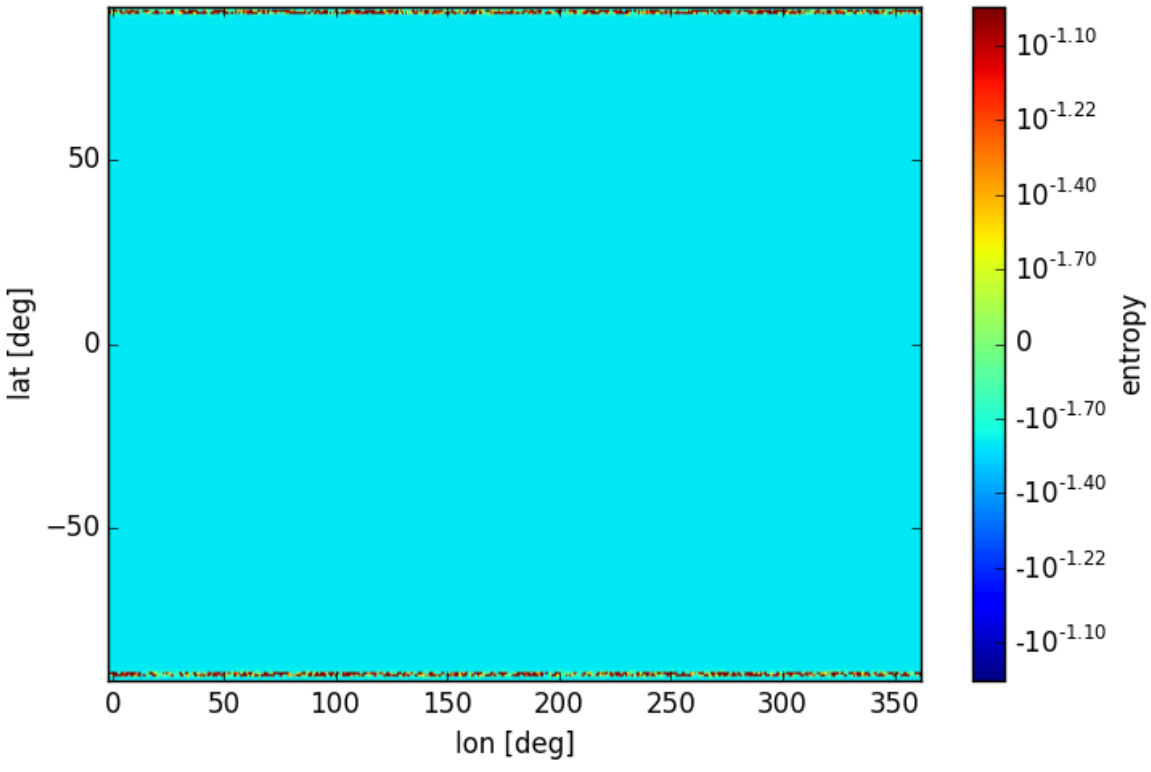
Millenium simulation - low res



$$dx = 0.006$$
$$d\theta = 0.037$$
$$d\phi = 0.049$$

$$dt = 2.77 \cdot 10^{-4}$$
$$\text{to} \quad 5.76 \cdot 10^{-4}$$

Millenium simulation - hi res

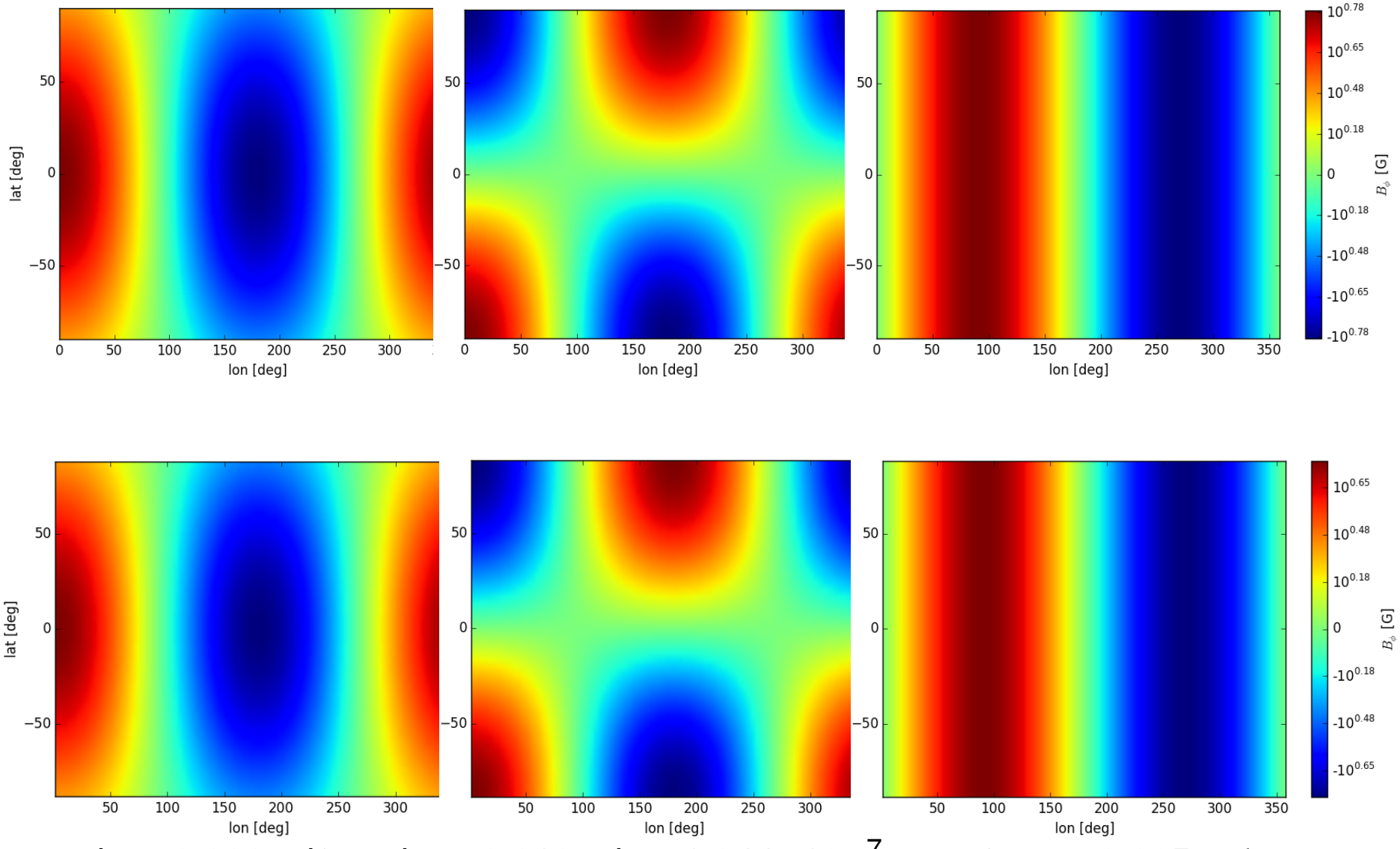


Millenium resolution:
NaNs!!

$$dx = 0.002$$
$$d\theta = 0.010$$
$$d\phi = 0.012$$

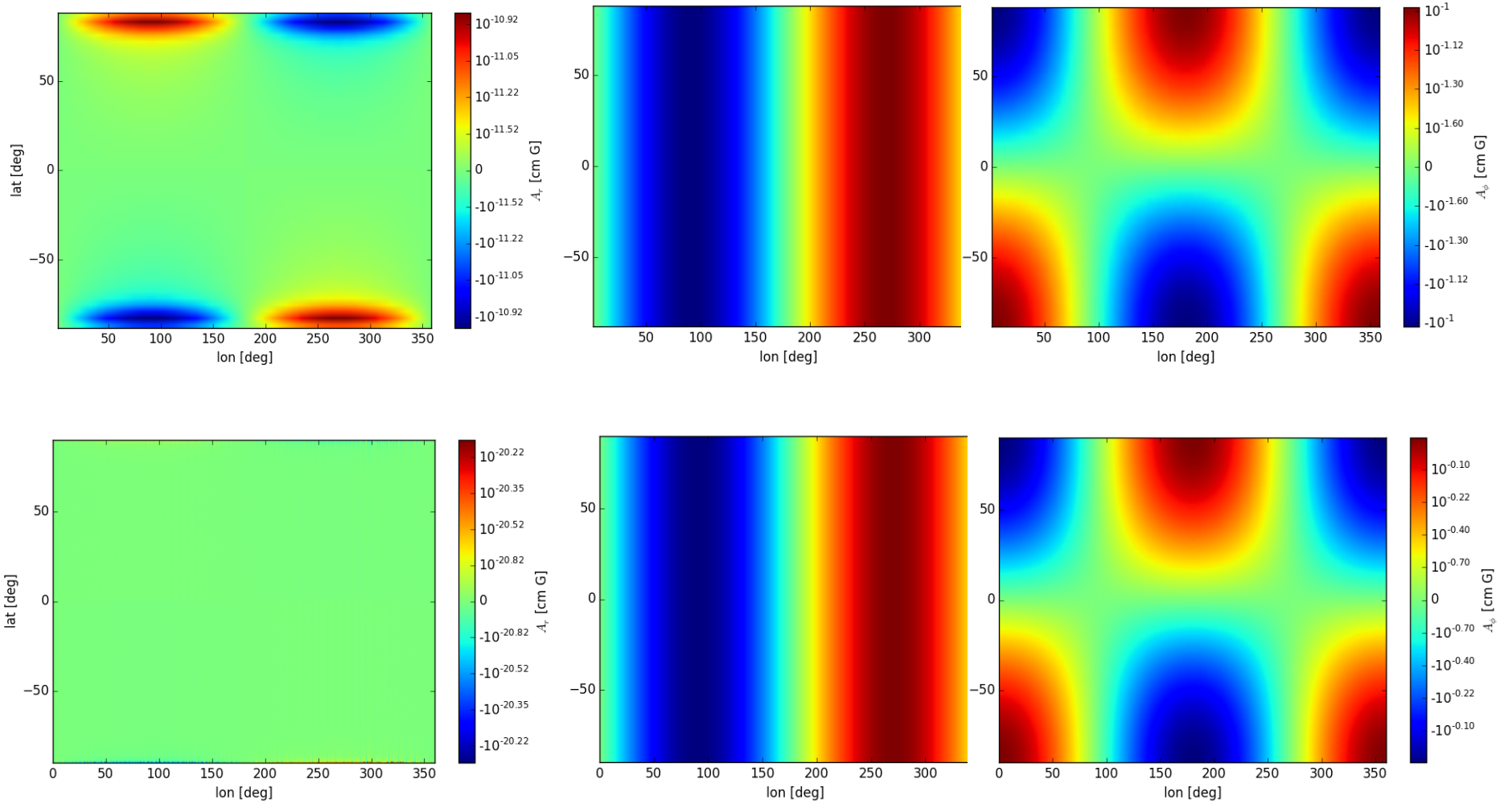
$$\nabla = \left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \theta}, \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi} \right)$$

Diffused magnetic field (low-res top)



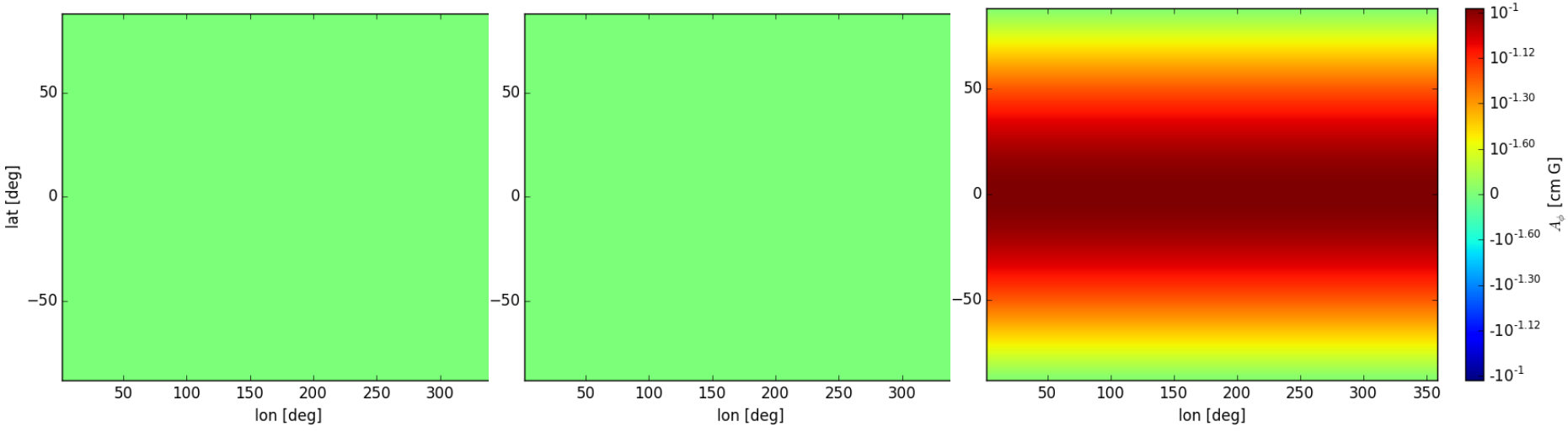
$dx = 0.002,\ d\theta = d\phi = 0.010,\ dt = 1.944 \cdot 10^{-7}$ i.e. $\theta_{\min} = 0.005$ radians

Magnetic potential resolution comparison



Low res (evolved longer) - radial component should remain zero

Time evolve vertical dipole



Note $A_\theta \equiv A_\phi \equiv 0$ in vertical dipole simulation!
Implies problem arises from

$$\frac{\partial A_r}{\partial t} \propto \frac{1}{r \sin\theta} \frac{\partial A_\theta}{\partial \phi} \text{ or } \frac{1}{r \sin\theta} \frac{\partial A_\phi}{\partial \phi} \quad \text{given } \frac{\partial A_\theta}{\partial \theta} \equiv 0 \text{ and not } \frac{1}{r} \frac{\partial A_\phi}{\partial \theta}$$

Implementation and checks

- θ-plane at the pole: MPI comm in the ϕ direction!

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- Increase time step once instability eliminated.