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Answer Set Programming as SAT modulo Acyclicity

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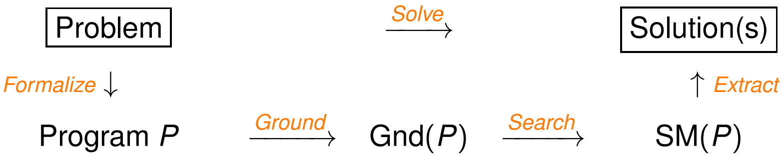
Answer Set Programming

Answer set programming (ASP) features a rule-based syntax subject to answer-set semantics.



Some native answer set solvers:

- CLASP `http://potassco.sourceforge.net/`
- CMODELS `http://www.cs.utexas.edu/~tag/cmodels/`
- DLV `http://www.dlvsystem.com/`
- IDP^3 `http://dtai.cs.kuleuven.be/krr/software/idp/`
- SMODELS `http://research.ics.aalto.fi/software/`

Example: SuDoku Puzzle

1	9	3	8	6	7	4	2	5
4	6	8	5	3	2	9	1	7
7	5	2	1	4	9	6	8	3
6	2	1	4	7	3	5	9	8
5	3	4	9	1	8	7	6	2
9	8	7	2	5	6	3	4	1
2	1	6	3	9	5	8	7	4
8	7	5	6	2	4	1	3	9
3	4	9	7	8	1	2	5	6

```
number(1..9).
border(1). border(4). border(7).
region(X,Y) :- border(X), border(Y).

1 { value(X,Y,N):number(X):number(Y):
    X1<=X: X<=X1+2: Y1<=Y: Y<=Y1+2 } 1
  :- number(N), region(X1,Y1).

:- 2 {value(X,Y,N):number(N)}, number(X), number(Y).
:- 2 {value(X,Y,N):number(Y)}, number(N), number(X).
:- 2 {value(X,Y,N):number(X)}, number(N), number(Y).
```

Example: Running the Solver

```
$ gringo sudoku.lp royle.lp | clasp 0
clasp version 3.0.4
Reading from stdin
Solving...
Answer: 1
value(1,3,2) value(1,9,1) value(2,2,7) value(2,5,3) value(3,5,4)
value(3,7,2) value(4,4,2) value(5,7,4) value(5,8,3) value(6,1,1)
value(6,3,5) value(6,4,6) value(7,5,7) value(8,2,3) value(9,4,1)
value(9,9,5) value(1,2,9) value(3,1,8) ...
Answer: 2
value(1,3,2) value(1,9,1) value(2,2,7) value(2,5,3) value(3,5,4)
value(3,7,2) value(4,4,2) value(5,7,4) value(5,8,3) value(6,1,1)
value(6,3,5) value(6,4,6) value(7,5,7) value(8,2,3) value(9,4,1)
value(9,9,5) value(3,1,9) ...
SATISFIABLE
Models       : 2
...
```

Key Features of ASP

- Typical ASP encodings follow a three-phase design:
  1. Generate the solution candidates
  2. Define the required concepts
  3. Test if a candidate satisfies its criteria
- Default negation favors concise encodings.
- Basic database operations are definable in terms of rules:
  - Projection: $\text{node}(X) \leftarrow \text{edge}(Y, X).$
  - Union: $\text{node}(X) \leftarrow \text{edge}(Y, X).$ $\quad \text{node}(Y) \leftarrow \text{edge}(Y, X).$
  - Intersection: $\text{symm}(X, Y) \leftarrow \text{edge}(X, Y), \text{edge}(Y, X).$
  - Complement: $\text{unidir}(X, Y) \leftarrow \text{edge}(X, Y), \text{not edge}(Y, X).$
- Recursive definitions are also supported:

$$\text{path}(X, Y) \leftarrow \text{edge}(X, Z), \text{path}(Z, Y), \text{node}(Y).$$

Translation-Based ASP

ASP can be implemented by translating ground programs into:

- Boolean Satisfiability (SAT)
  [J., ECAI, 2004; J. and Niemelä, MG-65, 2010]
- Integer Difference Logic (IDL)
  [Niemelä, AMAI, 2008; J. et al., LPNMR, 2009]
- Integer Programming (IP)
  [Liu et al., KR, 2012]
- Bit-Vector Logic (BV)
  [Nguyen et al., INAP, 2011; Extended in 2013]

☞ Existing solver technology can be harnessed for ASP!

Motivation

- Complexities of translations vary in program length n:

$\mathcal{O}(n)$	IDL, IP, BV	
$\mathcal{O}(n \times \log_2 n)$	SAT	[J., ECAI 2004]
$\mathcal{O}(n^2)$	SAT	[Lin & Zhao, IJCAI 2003]
$\mathcal{O}(2^n)$	SAT	[Lin & Zhao, AIJ 2004]

- What would be a minimal extension of SAT such that
  1. a linear embedding from ASP is enabled and
  2. the extension is efficiently implementable?
- In this paper, we consider embeddings into an extension based on graphs subject to an acyclicity constraint:

  M. Gebser, T. Janhunen, and J. Rintanen:
  "*Satisfiability Modulo Graphs: Acyclicity*" [JELIA 2014].

Outline

Formalisms of Interest

Translating Programs into SAT modulo Acyclicity

Implementation and Experiments

Conclusion

Source Formalism: Normal Programs

- Normal logic programs (NLPs) consist of rules of the form:

$$a \leftarrow b_1, \ldots, b_n, \text{not } c_1, \ldots, \text{not } c_m.$$

- The semantics is given by stable models, also known as answer sets, satisfying [Gelfond and Lifschitz, ICLP, 1988]:

$$M = \text{LM}(P^M).$$

Example

Consider the following program:

$$a \leftarrow b. \quad a \leftarrow c. \quad b \leftarrow a. \quad c \leftarrow \text{not } d. \quad d \leftarrow \text{not } c.$$

$\Longrightarrow$ $M_1 = \{a, b, c\}$ is stable but $M_2 = \{a, b, d\}$ is not.

Target Formalism: Syntax

A theory in SAT modulo acyclicity (ACYC) is a tuple $\langle X, C, N, A, l \rangle$ where

1. C is a set of clauses based on propositional variables in X,
2. $G = \langle N, A \rangle$ is a directed graph with a finite set of nodes N and arcs $A \subseteq N \times N$, and
3. $l : A \to X$ is a labeling that assigns a propositional variable $l(u, v)$ to every arc $\langle u, v \rangle \in A$ in the graph G.

Example

Rewriting our NLP using $N = \{a, b\}$ and $E = \{\langle a, b \rangle, \langle b, a \rangle\}$:

$$a \lor \neg b, \quad a \lor \neg c, \quad \neg a \lor b \lor c, \quad b \lor \neg a, \quad \neg b \lor a,$$
$$c \lor d, \quad \neg c \lor \neg d, \quad \neg a \lor c \lor e_{\langle a,b \rangle}, \quad \neg b \lor e_{\langle b,a \rangle}.$$

Target Formalism: Semantics

An ACYC theory $T = \langle X, C, N, A, l \rangle$ is satisfied by an interpretation $M \subseteq X$, denoted $M \models T$, iff

1. $M \models C$ and
2. $\langle N, A_M \rangle$ with $A_M = \{\langle u, v \rangle \in A \mid M \models l(u, v)\}$ is *acyclic*.

Example

Recall the theory T from our running example:

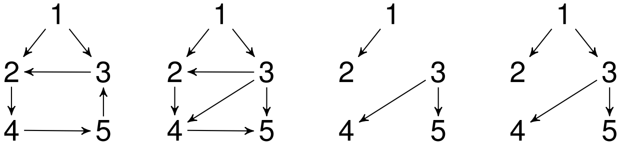
$$a \lor \neg b, \quad a \lor \neg c, \quad \neg a \lor b \lor c, \quad b \lor \neg a, \quad \neg b \lor a,$$
$$c \lor d, \quad \neg c \lor \neg d, \quad \neg a \lor c \lor e_{\langle a,b \rangle}, \quad \neg b \lor e_{\langle b,a \rangle}.$$

$$\Longrightarrow \quad M_1 = \{a, b, c, e_{\langle b,a \rangle}\} \models T \quad \text{but}$$
$$M_2 = \{a, b, d, e_{\langle a,b \rangle}, e_{\langle b,a \rangle}\} \not\models T.$$

Applications in Sight

Acyclicity constraints lend themselves for many purposes:

- Specifying a variety of topological structures:
  - Trees and forests (both directed and undirected)
  - Directed acyclic graphs (DAGs)
  - Chordal graphs



- Hamiltonian cycles
- Formalizing paths and reachability in general

General Translation from ASP to ACYC

- The classical models of the completion Comp(P) coincide with the supported models P [Apt et al., 1988].
- The strong groundedness of stable models can be captured by assigning numbers/ordinals to atoms [Elkan, AIJ 1990; Fages, JMLCS 1994; Erdem & Lifschitz, TPLP 2003].
- We follow the linear translation into IDL based on level rankings [Niemelä, AMAI 2008; J. et al., LPNMR 2009].
- The translation has to be applied only to atoms $a \in \mathrm{At}(P)$ having a non-trivial component $\mathrm{SCC}(a)$ with $|\mathrm{SCC}(a)| > 1$.

In our running example, we have $\mathrm{SCC}(a) = \{a, b\} = \mathrm{SCC}(b)$:

$$a \leftarrow b. \quad a \leftarrow c. \quad b \leftarrow a. \quad c \leftarrow \mathrm{not}\ d. \quad d \leftarrow \mathrm{not}\ c.$$

Identifying Rule Bodies

Following [Tseitin, 1968], the body $\mathrm{B}(r)$ of a defining rule $r \in \mathrm{Def}_P(a)$ is given a new name bd_r by

1. the clause $\mathrm{bd}_r \lor \bigvee_{b \in \mathrm{B}^+(r)} \neg b \lor \bigvee_{c \in \mathrm{B}^-(r)} c$,
2. for each $b \in \mathrm{B}^+(r)$, the clause $\neg \mathrm{bd}_r \lor b$, and
3. for each $c \in \mathrm{B}^-(r)$, the clause $\neg \mathrm{bd}_r \lor \neg c$.

$\Longrightarrow$ Effectively, we have $\mathrm{bd}_r \leftrightarrow \bigwedge_{b \in \mathrm{B}^+(r)} b \land \bigwedge_{c \in \mathrm{B}^-(r)} \neg c$.

Example

Rule:	$a \leftarrow b.$	$a \leftarrow c.$	$b \leftarrow a.$
Translation:	$\mathrm{bd}_1 \lor \neg b$	$\mathrm{bd}_2 \lor \neg c$	$\mathrm{bd}_3 \lor \neg a$
	$\neg \mathrm{bd}_1 \lor b$	$\neg \mathrm{bd}_2 \lor c$	$\neg \mathrm{bd}_3 \lor a$

Well Support from Internal Rules

For the well-support provided by a rule $r \in \mathrm{IDef}_P(a)$:

1. The clause $\mathrm{ws}_r \vee \neg\mathrm{bd}_r \vee \bigvee_{b \in \mathrm{B}^+(r) \cap \mathrm{SCC}(a)} \neg\mathrm{e}_{\langle a,b \rangle}$.
2. The clause $\neg\mathrm{ws}_r \vee \mathrm{bd}_r$.
3. For each $b \in \mathrm{B}^+(r) \cap \mathrm{SCC}(a)$, the clause $\neg\mathrm{ws}_r \vee \mathrm{e}_{\langle a,b \rangle}$.

$\Longrightarrow$ Effectively, we have $\mathrm{ws}_r \leftrightarrow \mathrm{bd}_r \wedge \bigwedge_{b \in \mathrm{B}^+(r) \cap \mathrm{SCC}(a)} \mathrm{e}_{\langle a,b \rangle}$.

Example

Internal rule:	$a \leftarrow b.$	$b \leftarrow a.$
Translation:	$\mathrm{ws}_1 \vee \neg\mathrm{bd}_1 \vee \neg\mathrm{e}_{\langle a,b \rangle}$ $\neg\mathrm{ws}_1 \vee \mathrm{bd}_1$ $\neg\mathrm{ws}_1 \vee \mathrm{e}_{\langle a,b \rangle}$	$\mathrm{ws}_3 \vee \neg\mathrm{bd}_3 \vee \neg\mathrm{e}_{\langle b,a \rangle}$ $\neg\mathrm{ws}_3 \vee \mathrm{bd}_3$ $\neg\mathrm{ws}_3 \vee \mathrm{e}_{\langle b,a \rangle}$

Enforcing Support for Atoms

For the definition $\mathrm{Def}_P(a)$ of an atom a in a program P:

1. For each $r \in \mathrm{Def}_P(a)$, the clause $a \vee \neg \mathrm{bd}_r$.
2. The clause $\neg a \vee \bigvee_{r \in \mathrm{EDef}_P(a)} \mathrm{bd}_r \vee \bigvee_{r \in \mathrm{IDef}_P(a)} \mathrm{ws}_r$.

$\Longrightarrow$ Effectively, this entails that $a \leftrightarrow \bigvee_{r \in \mathrm{Def}_P(a)} \mathrm{B}(r)$.

Example

Definition:	$a \leftarrow b.$ $\quad a \leftarrow c.$	$b \leftarrow a.$
Translation:	$a \vee \neg \mathrm{bd}_1,$ $\quad a \vee \neg \mathrm{bd}_2$ $\neg a \vee \mathrm{ws}_1 \vee \mathrm{bd}_2$	$b \vee \neg \mathrm{bd}_3$ $\neg b \vee \mathrm{ws}_3$

Overall Properties of the Translation

- The resulting translation $\mathrm{Tr}_{\mathrm{ACYC}}(P)$ of a normal program P is linear in the length of P.
- A one-to-many correspondence between the stable models of P and the models of $\mathrm{Tr}_{\mathrm{ACYC}}(P)$ is obtained.

Proposition

Let P be a normal logic program and $\mathrm{Tr}_{\mathrm{ACYC}}(P)$ its translation into SAT modulo acyclicity.

1. *If $M \in \mathrm{SM}(P)$, then there is a model $N \models \mathrm{Tr}_{\mathrm{ACYC}}(P)$ such that $M = N \cap \mathrm{At}(P)$.*
2. *If $N \models \mathrm{Tr}_{\mathrm{ACYC}}(P)$, then $M \in \mathrm{SM}(P)$ for $M = N \cap \mathrm{At}(P)$.*

Extension: Disabling Edges Dynamically

- An edge variable $e_{\langle a,b \rangle}$ can be falsified if
  1. a is known to be false,
  2. a has an externally supporting rule, or
  3. a has an internally supporting rule $r \in \mathrm{IDef}_P(a)$ such that $b \notin \mathrm{B}^+(r)$.
- The extended translation $\mathrm{Tr}^+_{\mathrm{ACYC}}(P)$ gives rise to a similar but tighter correspondence of models.

Example

Definition:	$a \leftarrow b. \quad a \leftarrow c.$	$b \leftarrow a.$
Case 1:	$a \vee \neg e_{\langle a,b \rangle}$	$b \vee \neg e_{\langle b,a \rangle}$
Case 2:	$\neg bd_2 \vee \neg e_{\langle a,b \rangle}$	–
Case 3:	–	–

Implementation

- For tool interoperability, the SMODELS format is used as an intermediate format for representing ground programs.
- Extended rules, such as choice, cardinality, and weight rules may have to be translated away using LP2NORMAL2.
- To enable cross-translation for different back-end solvers,
  1. the input program is instrumented with auxiliary atoms and auxiliary rules corresponding to $\mathrm{Tr}^{+}_{\mathrm{ACYC}}$ and
  2. the completion is produced in the target format of interest.
- Our tools produce a number of output formats:
  1. DIMACS with optional ACYC and MAXSAT extensions
  2. SMT Library 2.0 (QF_IDL and QF_BV fragments)
  3. PB format
  4. CPLEX

Tool Support

<table>
<tr><td colspan="2">gringo / lparse</td></tr>
<tr><td colspan="2">lpstrip</td></tr>
<tr><td colspan="2">lpcat</td></tr>
<tr><td>lp2normal2</td><td>–</td></tr>
<tr><td colspan="2">lp2acyc</td></tr>
<tr><td>lp2sat
[-g]</td><td>acyc2solver
[--diff]
[--bv]
[--pb]
[--mip]</td></tr>
</table>

The tool collection is published under:

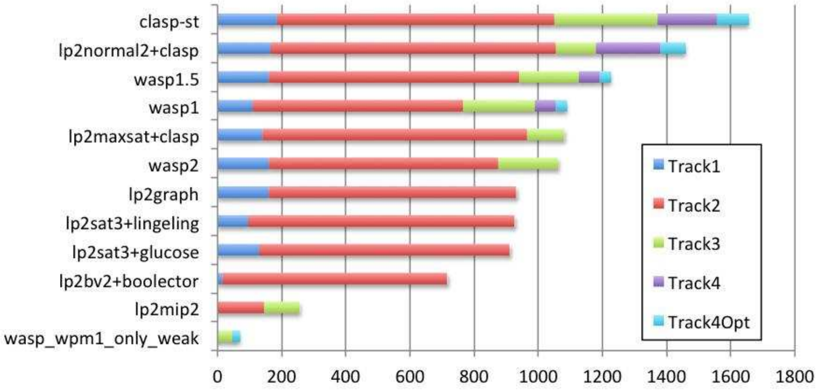
`http://research.ics.aalto.fi/software/asp/lp2acyc/`

Experiments

Problem	Hamilton		Tree			
Size	100	150	25	50	75	100
CLASP	0.95	20.16	4.37	1193.09	1495.32	1995.19
ACYCGLUCOSE	0.07	0.15	**0.74**	315.83	999.07	1414.68
ACYCMINISAT	**0.04**	**0.12**	0.83	544.43	1025.02	1224.28
Z3	2.45	50.64	4.75	1208.36	1726.56	2538.20
ACYCGLUCOSE-Tr_{ACYC}	0.93	13.75	1.40	271.93	973.22	1388.82
ACYCMINISAT-Tr_{ACYC}	0.76	7.28	0.80	484.92	879.18	1030.79
Z3-Tr_{ACYC}	35.80	331.11	6.30	1178.44	2266.66	2714.01
ACYCGLUCOSE-Tr^{+}_{ACYC}	**0.04**	0.18	1.09	**264.28**	931.28	1379.15
ACYCMINISAT-Tr^{+}_{ACYC}	0.08	0.32	0.77	473.64	**852.78**	**1016.50**
Z3-Tr^{+}_{ACYC}	27.72	239.83	7.03	1230.51	1976.20	2562.70

ASP Competition 2014

The LP2GRAPH system was based on the translation $\mathrm{Tr}^{+}_{\mathrm{ACYC}}$ and using ACYCGLUCOSE as the back-end solver.



[https://www.mat.unical.it/aspcomp2014/]

Conclusion

- Translation-based ASP aims to exploit
  - the expressive power of ASP and
  - the potential behind existing solver technology.
- The translation from ASP into SAT modulo acyclicity
  - is linear and
  - preserves stable models up to original signature.
- The cross-translation of ASP is enabled by
  - a suitable intermediate format and
  - postponing format-specific aspects to the last step.
- Future extensions:
  - Support for further formats and solver types
  - Covering optimization more widely